A Nonlinear ILC Schemes for Nonlinear Dynamic Systems To Improve Convergence Speed

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Abstract

In this paper we study nonlinear Iterative Learning Control (ILC) schemes for nonlinear dynamic systems. The methods are proposed in this paper such as Newton-Type and Secant-Type, for improve the convergence speed in nonlinear ILC schemes systems. In nonlinear Iterative Learning Control methods convergence speed is faster than the linear ILC schemes systems. The system taken into consideration is the one phase control of the switched reluctance motor (SRM) and simulation result show that the convergence speed of the Newton-Type ILC schemes is the fastest and the Secant-Type is faster than the linear-type but slower than the Newton-Type. On the other hand nonlinear ILC schemes require more prior knowledge about the system.

Keywords*: *ILC, Newton-Type, Secant-Type, Convergence Speed.

1. Introduction

The convergence speed of the linear-type ILC scheme is approved by a positive steady ration (less than one) of tracking errors between two repeated iterations. This is due to the use of linear- type updating law with a constant learning gain which is iteration-dependent. Some techniques in Numerical Analysis have been used in the learning control design [1, 2]. Recently the Newton-type ILC [3] and Quasi-Newton type ILC schemes [4] have been proposed and explored targeting at achieve a faster convergence speed. In this paper we extended the results to the general dynamic systems [5].

In order to improve the convergence speed, the Newton-type ILC scheme is first presented in the area of iterative learning control to provide a corresponding method to linear-type ILC schemes [5, 6]. A nonlinear gain takes the place of constant gain and is proven to be able to speed up the learning convergence speed [7, 8]. However the faster convergence is achieved at the price of the need for more prior information. The linear-type ILC only used the system output signals, while the Newton-type ILC needs the biased derivative of the system input-output mapping.

The Secant-type ILC scheme, on the other hand, approximates the Newton-type ILC scheme show that the convergence speed of the Secant-type ILC scheme is between the linear-type ILC scheme and the Newton-type ILC scheme. As well known, both the Newton and Secant methods, in Numerical Analysis, can only guarantee a faster convergence speed around the stability, i.e., a restrictive convergence condition is required [9-11]. In practice, this condition can hardly be met. In order to extend the convergence range of learning, the linear-type ILC scheme is employed when the output of the dynamic system is outside a particular error bound [12, 13]. When inside the tracking error bound, the Newton-type ILC scheme or Secant-type ILC scheme will take over the learning, expedition the learning speed considerably while retaining the learning convergence property[14,15].

This paper is organized as follows: The problem is formulated in section 2. The Newton-type ILC scheme is presented in section3. In section 4, the Secant-type scheme is addressed. Section 5 provides an illustrative example for comparison. Then conclusion follows in section 6.

2. Problem Statement

Let us look at the following dynamic system

$$\dot{x}(t) = f(X(t), u(t), t) \quad x(0)=x_0 \qquad (1)$$
$$y(t) = g(X(t), u(t), t),$$

Where $X \in \chi, u \in U$ and $t \in [0,T]$ is a compact subset of R^n and U a compact separation of R, The dynamic system (1) is repeatable over [0, T], and satisfies the following assumptions:[1]

Assumption 1: The vector-valued f(x, u, t) and scalar g(x, u, t) are at least twice continuously differentiable in the compact set $\Omega = \chi \times U \times [0,T]$ with respect to x, u and t correspondingly.

Remark1. From the continuity of f and g in Assumption 1, the following results can be obtained directly:

1. Nonlinear function f(x, u, t) and its firsts derivatives with respect to x and u are bounded in the compact set Ω, therefore, there exists a constant f_0 such that:

$$\| f(x_1,u_1,t) - f(x_2,u_2,t)\| \le f_0\left(\|x_1 - x_2\| + |u_1 - u_2|\right) \qquad (2)$$

2. Nonlinear function g(x, u, t) and its first derivatives with respect to x and u are bounded in the compact set Ω, therefore, there exists a constant g_0 such that:

$$\|g(x_1,u_1,t) - g(x_2,u_2,t)\| \le g_0\left(\|x_1 - x_2\| + |u_1 - u_2|\right) \qquad (3)$$

3. Denote

$$g_{xx}(t) \triangleq \frac{\partial^2 g(x(t),u(t),t)}{\partial x^2}, \quad g_{xu}(t) \triangleq \frac{\partial^2 g(x(t),u(t),t}{\partial x \partial u},$$

$$g_{uu}(t) \triangleq \frac{\partial^2 g(x(t),u(t),t)}{\partial u^2}$$

Where $\|g_x\|, |g_{uu}|, \|g_{xu}\|$ and $\|g_{xx}\|$, are bounded by some finite constants M_x, M_{uu}, M_{xu} and M_{xx} respectively in the compact set Ω.

Remark2.Consider Assumption 1, is more restrictive as it requires not only Lipschitz continuity condition for both f (0) and g (0) but also the boundedness of the second derivatives for both f (0) and g (0). It can be seen later that, such a restrictive requirementis needed in order to achieve a faster convergence speed.

Assumption2. It is assumed that:

$$0 < \alpha_1 \le \frac{\partial g}{\partial u} \le \alpha_2, \forall (x,u,t) \in \Omega.$$

Remark3. g_u Represents the system gain, which could be nonlinear in **Ω.**

Assumption3. The identical initialization condition holds for all iterations, i.e. $x_i(0) = x_0, \forall i \in Z$

3. Convergence Analysis for Linear-Type ILC Scheme

The convergence analysis for linear-type ILC scheme given with assumptions i.e. the uniqueness of $u_d(t)$, in the following we show that, without the requirement of such an assumption, the linear-type ILC scheme still works [8].

Theorem1. For the dynamic system (1), associated with the desired out-put $y_d(t)$ and the linear-type ILC scheme, the monotonic convergence of with the time-weighted norm is strictly guaranteed in the iteration domain if (2) is satisfied

Proof:

Denote

$$\Delta_i x \triangleq x_{i+1} - x_i, \Delta_i u \triangleq u_{i+1} - u_i, \xi_i \triangleq (x_i + v\Delta_i x, u_i + v\Delta_i u, t)$$

where $0 \le v \le 1$.

Applying Taylor's theorem, $\forall t \in [0,T]$, we have

$$\begin{aligned} g(x_{i+1},u_{i+1},t) &= g(x_i + \Delta_i x, u_i + \Delta_i u, t) \\ &= g(x_i,u_i,t) + q_1 \Delta y_i g_u(\xi_i) + \Delta_i x^T g_x(\xi_i). \\ &= y_i(t) + q_1 \Delta y_i g_u(\xi_i) + \Delta_i x^T g_x(\xi_i) \end{aligned} \qquad (4)$$

The tracking error at the (i+1)th iteration is:

$$\begin{aligned} \Delta y_{i+1} &= \Delta y_i + y_i - y_{i+1} \\ &= \Delta y_i - q_1 \Delta y_i g_u(\xi_i) - \Delta_i x^T g_x(\xi_i) \\ &= [1 - q_1 g_u(\xi_i)]\Delta y_i + [x_i - x_{i+1}]^T g_x(\xi_i) \end{aligned} \qquad (5)$$

According to Assumption 1 and Assumption 3, it follows that:

$$x_i - x_{i+1} = \int_0^t [f(x_i,u_i,t) - f(x_{i+1},u_{i+1},t)]d\tau$$

Taking norm to both sides of the above equation, noticing the Lipschitz continuity condition of f(x (t), u (t), t), it follows that:

$$\begin{aligned} \|x_i - x_{i+1}\| &\le f_0 \int_0^t (\|x_i - x_{i+1}\| + |u_i - u_{i+1}|)d\tau \\ &\le f_0 \int_0^t \left(\|x_i - x_{i+1}\| + |u_i - u_{i+1}|\right)d\tau \\ &\le f_0 \int_0^t \left(\|x_i - x_{i+1}\| + q_1|\Delta y_i|\right)d\tau \end{aligned} \qquad (6)$$

Applying Gronwall Lemma:

$$\begin{aligned} \|x_i - x_{i+1}\| &\le f_0 e^{f_0 t} \int_0^t q_1 |\Delta y_i| d\tau \\ &\le f_0 e^{f_0 t} q_1 \int_0^t e^{\lambda\tau} d\tau |\Delta y_i|_\lambda \\ &= f_0 e^{f_0 t} q_1 \frac{e^{\lambda t} - 1}{\lambda} |\Delta y_i|_\lambda \end{aligned} \qquad (7)$$

Hence

$$\|x_i - x_{i+1}\|_\lambda = \max_{t\in[0,T]} e^{-\lambda t}\|x_i - x_{i+1}\|$$
$$\le \max_{t\in[0,T]} f_0 e^{f_0 t} q_1 \frac{1-e^{-\lambda t}}{\lambda}|\Delta y_i|_\lambda \qquad (8)$$
$$= f_0 e^{f_0 T}\frac{1-e^{-\lambda T}}{\lambda}|q_1||\Delta y_i|_\lambda$$

Taking norm of (5) yields:

$$|\Delta y_{i+1}| \le |1 - q_1 g_u(\xi_i)||\Delta y_i| + M_x\|x_i - x_{i+1}\|.$$

Taking the time-weighted norm of the above inequality, according to Assumption 2 and (8), we have:

$$|\Delta y_{i+1}|_\lambda \le \left\{|1-q_1 g_u(\xi_i)|_s + M_x|q_1| f_0 e^{f_0 T}\frac{1-e^{-\lambda T}}{\lambda}\right\}|\Delta y_i|_\lambda \qquad (9)$$

We define $\gamma_1 = |1-q_1 g_u(\xi_i)|_s$, the corresponding δ_1 equals to $M_x|q_1| f_0 e^{f_0 T}\frac{1-e^{-\lambda T}}{\lambda}$, (9) can be written as:

$$|\Delta y_{i+1}|_\lambda \le (\gamma_1 + \delta_1)|\Delta y_i|_\lambda \qquad (10)$$

With a sufficiently large $\lambda \ge \lambda_1$, the linear-type ILC scheme convergence and the convergence speed is:

$$Q(L_1, 0) = \lim_{i\to\infty} \sup \frac{\|\Delta y_{i+1}\|_\lambda}{\|\Delta y_i\|_\lambda} = \gamma_1 + \delta_1 < 1 \qquad (11)$$

This concludes the proof.

Remark4. It is obvious that the input sequence and output sequence have the same convergence speed. Therefore, the fastest convergence speed can be achieved in the presence of uncertainty g_u, by solving the min-max problem. The optimal gain q_1 is $\frac{2}{\alpha_2+\alpha_1}$ with the fastest convergence speed $J_1 = \theta + \frac{2}{\alpha_2+\alpha_1}c(\lambda)$.

Remark5. The fastest convergence speed depends on two parameters α_1 and α_2. Even the optimal technique is employed, if $\alpha_2 \gg \alpha_1$, $J_1 \to 1$ i.e. the learningconvergence will be extremely slow. On the other hand, if g_u is available, according to (9) we can choose $q_1 = \frac{1}{g_u}$, which can bring J_1 to the lowest level. This simple observation motivates us to develop the Newton-type ILC scheme which can considerably improve the convergence speed.[1]

4. The Newton-Type ILC Scheme

A fixed learning gain q_1 is utilized in the linear-type ILC approach, which limits the learning convergence rate. A fast convergence is always preferred in real applications. A smaller Q-factor implies a faster convergence rate. The Newton-type approach is well known in Numerical Analysis as it guarantees a fast convergence speed for memory-less iterative process. The Newton-type ILC approach, originated from the same idea, is introduced in the design of iterative learning control law for nonlinear non-affine dynamic systems, aiming at improving the convergence speed of the learning process in the neighbourhood of the desired trajectory. In the static iterative process, the Newton-type scheme guarantees a fast convergence speed [3]. For the dynamic iterative process, since a nonlinear learning factor q_1 is used, Newton-type ILC can also achieve a much faster convergence speed than that of the linear-type ILC scheme in the sense of Q-factor [4].

Note that Q-factor of convergence is a local concept. In order to widen the convergence range of learning, the linear-type ILC scheme is employed when the output of the dynamic system is outside a specified error bound. When inside the tracking error bound, the Newton-type ILC scheme will take over the learning job, improve the learning speed significantly while retaining the learning convergence property [5].

Suppose $g_u(t)$ is known. From Assumption 2 $0 < \alpha_1 \le g_u(t) \le \alpha_2 \quad \forall(\text{x,u,t}) \in \Omega$. The Newton-type ILC scheme is constructed as

$$L_N : u_{i+1}(t) = \begin{cases} u_i(t) + q_1\Delta y_i(t) \text{ if } \|\Delta y_i\|_s \ge \frac{2\theta\alpha_1^2}{M_{uu}} \\ u_i(t) + \frac{\Delta y_i(t)}{g_{u,i}(t)} \quad \text{else} \end{cases} \qquad (12)$$

Where L_N stands for the Newton-type iterative process, $q_1 = \frac{2}{\alpha_1+\alpha_2}$ is the robust optimal learning gain for the linear-type ILC, $\theta = \frac{\alpha_2-\alpha_1}{\alpha_2+\alpha_1}$ and

$$\Delta y_i(t) = y_d(t) - y_i(t)$$

Theorem2. For the dynamic system (1), the control updating law (12) will make the tracking error $\|\Delta y_i\|_s \to 0$ as $\text{i}\to\infty$. Newton-type ILC is Q-faster than linear-type ILC scheme in the sense that

$$Q(L_N, 0) \ll Q(L_1, 0)$$

Remark6. Let us explain the Newton-type ILC scheme further by a linear time-invariant system

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

Where $g_u = d$ is constant. If $q_1 = \frac{1}{g_{u,i}} = \frac{1}{d}$, which leads to $|1 - q_1 q_2| = 0$, the convergence speed is the fastest.

5. The Secant-Type ILC Scheme

Secant algorithm in Numerical Analysis provides a good approximation to Newton algorithm, which motivates the introduction of the Secant-type ILC schemes.

It should be noted that Secant method in Numerical Analysis is also a local concept, the tracking error converge only when the output of the system at first iteration is near the desired trajectory. In this section, the Secant-type ILC approach is constructed for the non-linear non-affine dynamic system in the sense of global convergence: when the output of the dynamic system is far away from the desired trajectory, the linear-type ILC approach is employed to guarantee the convergence of the desired trajectory, the Secant-type ILC approach is employed to enhance the convergence rate.[1]

The Secant-type ILC scheme is constructed as:

$$L_s:$$

$$u_{i+1}(t) = \begin{cases} u_i(t) + q_1 \Delta y_i(t) & else \\ u_i(t) + \dfrac{\Delta y_i(t)(u_i(t) - u_{i-1}(t)}{g(x_i, u_i, t) - g(x_i, u_{i-1}, t)} & if \ \|\Delta y_i\|_s < \dfrac{2(\theta - k_1)\alpha_1^2}{M_{uu}} \\ & \& |u_i - u_{i-1}, t| < \alpha \end{cases} \quad (13)$$

$$\alpha \triangleq \frac{k_1 \alpha_1}{M_{uu}}$$

Where L_s stands for the Secant-type iterative process, and $k_1 < \theta$ is a finite constant.

Remark7. It can be seen that Secant-type ILC is a second order scheme, but nonlinear in nature.

Theorem3. For the dynamic system (1), the control updating law will make the tracking error $\|\Delta y_i\|_s \to 0$ as $i \to \infty$. The Secant-type ILC is Q-faster than linear-type ILC scheme, however slower than the Newton-type ILC scheme in the sense that

$$Q(L_N, 0) \ll Q(L_s, 0) < Q(L_1, 0)$$

This shows that the convergence speed of the Secant-type ILC scheme is in between the Newton-type ILC and the linear-type ILC schemes.

Remark8. In above comparison of the learning convergence speed, it is necessary to choose the maximum λ satisfying all requirements from the linear-type, Newton-type and Secant-type schemes. Under such time-weighted norm, the convergence speeds are compared.

6. Illustrative Example

Consider one phase control of the Switched Reluctance Motor model as follow:

$$\begin{aligned} &\frac{dx_1}{dt} = N_r x_2 \\ &J\frac{dx_2}{dt} = \sum_{j=1}^{2} T_j(x_1, u_j) \\ &T_j(x_1, u_j) = \frac{N_r \Psi_s}{h_j^2(x_1)} \frac{dh_j(x_1)}{dx_1} \left\{1 - \left[1 + u_j h_j(x_1)\right] e^{-u_j h_j(x_1)}\right\} \\ &h_j(x_1) = L_a + L_u \sin\left[x_1 - (j-1)2\pi/4\right] \end{aligned} \quad (14)$$

That is, only one phase torque is generated to follow the desired $y_d = 0.044$ Nm.

6.1. Linear-type ILC Scheme

First we choose the linear-type ILC scheme to track the desired output y_d. Choose $u_0(t) = 1$. To guarantee the convergence of the scheme, we choose $q_1 = \frac{2}{\alpha_1 + \alpha_2} = 13.92, and \ \theta = 0.4175$. Moreover

$$0 < |g_{uu}| = 0.0495, \forall (x, u) \in \Omega$$

The simulation result is shown in Fig 1(solid line). From the figure it can be seen that the linear-type ILC scheme generates a monotonous convergence.

The relative error $\max_{t \in [0,1]} \left|\frac{\Delta y_i}{y_d}\right|$ drops from 1 to 10^{-6} within 5 iterations. Further more, the curve of the error is very close to a straight line which indicates that the linear-type ILC scheme has the linear convergence order (geometric convergence in the log scale).

6.2. Newton-type ILC Scheme

For the dynamic system (14), The simulation result of the Newton-type ILC scheme is also shown in Fig1 (dotted line).When the tracking error is outside the error bound $\frac{2\theta\alpha_1^2}{M_{uu}}$, the convergence of Newton-type is the same as the linear-type ILC. When the tracking error enters the error bound, the tracking error converges in a much faster. After 6 iterations, the relative tracking error drops from 10^{-4} to 10^{-12}. From Fig1, it can be seen that the tracking error of the linear-type ILC scheme takes 17 iterations to reach the same precision level. Due to the computation precision such as quantization or finite word length, convergence speed ceases after the tracking error reaches 10^{-11}.

6.3. Secant-type ILC Scheme

For the dynamic system (14), the simulation result of Secant-type ILC scheme is also shown in Fig1 (dash-dotted line). Let $k_1 = 0.25 < \theta$, where the tracking error is outside the error bound $\frac{2(\theta - k_1)\alpha_1^2}{M_{uu}}$, the convergence of the Secant-type is the same as the linear-type ILC. When the tracking error enters the error bound, the convergence speed of the tracking error is in between the linear-type ILC and Newton-type ILC schemes. From Fig1, it can be seen that the tracking error of the Secant-type ILC scheme takes 12 iterations to reach the precision level of 10^{-11}.

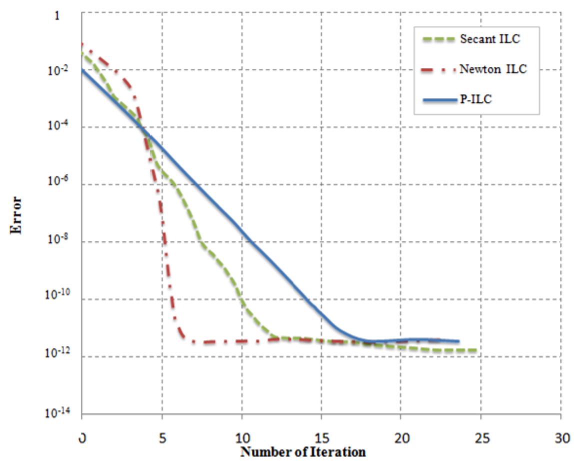

Fig.1 Convergence speed of the three ILC schemes for SRM

7. Conclusion

In this paper we studied nonlinear-type ILC schemes for nonlinear dynamic systems. The proposed nonlinear methods such as Newton-type and Secant-type ILC scheme improve the learning convergence speed. The convergence order of nonlinear ILC schemes is evaluated in an systematic method. It is concluded that the convergence speed of Newton-type ILC scheme is the fastest. The Secant-type is faster than that of the linear-type but slower than the Newton-type.

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