Simulated Annealing Clustering for Optimum GPS Satellite Selection

M. Ranjbar[1] and M. R. Mosavi[2]

**[1] Department of Computer Engineering, Iran University of Science and Technology
Narmak, Tehran 16846-13114, Iran**

**[2] Department of Electrical Engineering, Iran University of Science and Technology
Narmak, Tehran, 16846-13114, Iran
Corresponding Author**

Abstract

This paper utilizes a clustering approach based on Simulated Annealing (SA) method to select optimum satellite subsets from the visible satellites. Geometric Dilution of Precision (GDOP) is used as criteria of optimality. The lower the values of the GDOP number, the better the geometric strength, and vice versa. Not needing to calculate the inverse matrix, which is time-consuming process, is a dramatically important advantage of using this method, so a great reduction in computational cost is achieved. SA is a powerful technique to obtain a close approximation to the global optimum for a given problem. The evaluation of the performance of the proposed method is done by validation measures. The external validation measures, entropy and purity, are used to measure the extent to which cluster labels affirm with the externally given class labels. The overall purity and entropy is 0.9015 and 0.3993, respectively which is an excellent result.

Keywords: *GPS, GDOP, Clustering, Simulated Annealing.*

1. Introduction

The Global Positioning System (GPS) is a satellite-based navigation system that was developed by the U.S. Department of Defence (DOD) in the early 1970s [1]. GPS consists of constellation of at least 24 operational satellites. There are always at least four satellites visible, so four satellites are enough to provide positioning or location information.

The Geometric Dilution of Precision (GDOP) is a number which is a measure of the quality you might expect from a position measurement of the GPS system based solely on the geometric arrangement of the satellites and the receiver being used for the measurement [2]. Because of GDOP provides a simple interpretation of how much positioning precision can be diluted by a unit of measurement error, it is desirable to choose the combination of satellites in a satellite constellation with GDOP as small as possible. Several methods based on GDOP have been proposed to improve the GPS positioning accuracy [3]. Most, if not all, of those methods need matrix inversion to calculate GDOP. Though they can guarantee to achieve the optimal subset, the computational complexity is usually too intensive to be practical.

Instead of directly solving the GDOP equations and avoiding the time-consuming process of matrix inversion, rephrasing the GDOP calculation as approximation problems and employing NNs to solve such problems proposed in references [4-6]. However, solving classification and approximation problems using NN usually suffer from the long training time and difficulty in determining the NN architecture. The method employed in this paper is a solution to all of these problems.

Simulated Annealing (SA) is an optimization algorithm obtained from the physical process of cooling molten material down to the solid state [7]. It searches for the minimum energy state of the objective function without considering the shape of the function and can escape from local minima with hill-climbing [8]. Due to these features, SA has been widely used for different combinatorial and other optimization problems [9]. SA was applied in the proposed model.

The remainder of the paper is organized as follows. In section II, a brief review of GDOP computation is discussed. In section III, we discussed the clustering analysis problems. This is followed by introducing SA method, which is employed in this paper for clustering, in section IV. Section V shows the results of computer simulation. The paper is ended with the conclusions of our study in section VI.

2. Geometric Dilution of Precision

In GPS applications the GDOP is often used to select a subset of satellites from all visible ones. In order to determine the position of a receiver, pseudo-ranges from $n \geq 4$ satellites must be used at the same time. By linearizing the pseudo-range equation with Taylor's series expansion at the approximate (or nominal) receiver position, the relationship between pseudo-range difference (Δr_i) and positioning difference (Δx_i) can be summarized as follows [10]:

$$\begin{bmatrix}\Delta r_1\\ \Delta r_2\\ \vdots\\ \Delta r_n\end{bmatrix}=\begin{bmatrix}e_{11} & e_{12} & e_{13} & 1\\ e_{21} & e_{22} & e_{23} & 1\\ e_{31} & e_{32} & e_{33} & 1\\ \vdots & \vdots & \vdots & \vdots\\ e_{n1} & e_{n2} & e_{n3} & 1\end{bmatrix}\begin{bmatrix}\Delta x_1\\ \Delta x_2\\ \Delta x_3\\ c\Delta x_4\end{bmatrix}+\begin{bmatrix}v_1\\ v_2\\ \vdots\\ v_n\end{bmatrix} \quad (1)$$

which is written in a compact form as:

$$z = Hx + v \quad (2)$$

The $n\times 4$ matrix H is formed with direction cosines e_{i1}, e_{i2} and e_{i3} from the receiver to the ith satellite and v denotes a random noise with an expected value of zero. The difference between the estimated and true receiver positions is given by:

$$\tilde{x} = (H^T H)^{-1} H^T v \quad (3)$$

Where H^T denotes the transpose of H and $M = H^T H$, called the measurement matrix, is a 4×4 matrix no matter how large n is. It can be shown that the measurement matrix is symmetric and non-negative and thus it has four non-negative eigenvalues. Assuming $E\{vv^T\} = s^2 I$, then

$$E\{\tilde{x}\tilde{x}^T\} = (HH^T)^{-1} \quad (4)$$

The GDOP factor is defined as the square root of the trace of the inverse measurement matrix:

$$GDOP = \sqrt{trace(M)^{-1}} = \sqrt{\frac{trace[adj(M)]}{\det(M)}} \quad (5)$$

Because of GDOP provides a simple interpretation of how much positioning precision can be diluted by a unit of measurement error, it is desirable to choose the combination of satellites in a satellite constellation with GDOP as small as possible. Table 1 shows DOP ratings.

Table 1: DOP ratings

Class Number	DOP Value	Rating
Class 1	1-2	Excellent
Class 2	2-5	Good
Class 3	5-10	Moderate
Class 4	10-20	Fair
Class 5	>20	Poor

M is a 4×4 matrix so it has four eigenvalues, $l_i (i = 1,2,3,4)$. It is known that the four eigenvalues of M^{-1} is l_i^{-1}. we know that the trace of a matrix is the sum of its eigenvalues [5]. So equation (5) can be represented as:

$$GDOP = \sqrt{l_1^{-1} + l_2^{-1} + l_3^{-1} + l_4^{-1}} \quad (6)$$

By defining the four variables the mapping is performed as follows:

$$f_1(\vec{l}) = l_1 + l_2 + l_3 + l_4 = trace(M) \quad (7)$$

$$f_2(\vec{l}) = l_1^2 + l_2^2 + l_3^2 + l_4^2 = trace(M^2) \quad (8)$$

$$f_3(\vec{l}) = l_1^3 + l_2^3 + l_3^3 + l_4^3 = trace(M^3) \quad (9)$$

$$f_4(\vec{l}) = l_1 l_2 l_3 l_4 = \det(M) \quad (10)$$

GDOP can be considered as a functional $R^4 \to R^1$ mapping from $\vec{f}$ to GDOP with the inputs f_1, f_2, f_3, f_4 and output GDOP. Because of the non-linearity of mapping from $\vec{f}$ to GDOP, it is not possible to determine it analytically. So clustering algorithm is a good alternative.

3. The Clustering Problem

Clustering is the art of finding natural groupings or clusters in data based on some similarity measures. Let $O = \{o_1,...,o_n\}$ be a set of n patterns and let $A_{n\times d}$ be the pattern matrix with n rows and d columns. Each ith pattern is characterized by a real value d dimensional profile vector $a_i (i = 1,...,n)$, where each element a_{ij} corresponds to the jth real value feature $(j = 1,...,d)$ of the ith pattern $(i = 1,...,n)$.

Let $W = [w_{ig}](i = 1,...,n; g = 1,...,k)$ be the $n\times k$ cluster membership matrix where:

$$w_{ig} = \begin{cases}1 & ; \text{ if pattern } i \text{ is assigned to cluster g}\\ 0 & ; \text{ o.w.}\end{cases}$$

Let $Z = [z_{gj}](g = 1,...,k; j = 1,...,d)$ be a $k\times d$ matrix of cluster centers where:

$$z_{gj} = \frac{\sum_{i=1}^{n} w_{ig} a_{ij}}{\sum_{i=1}^{n} w_{ig}} \quad (11)$$

Given $A_{n\times d}$ the goal of a partitional clustering algorithm is to determine a partition $Z = \{z_1, z_2,..., z_k\}$ $(i.e., z_g \neq \Phi, \forall g; z_g \cap z_h = \Phi, \forall g \neq h; \cup_{g=1}^{k} z_g = O)$ such that patterns which belong to the same cluster are as similar to each other as possible, while patterns which belong to different clusters are as dissimilar as possible. The sum of squared Euclidean distance [11] is the most popular objective function for continuous features which should be minimized:

$$J(W,Z) = \sum_{i=1}^{n}\sum_{g=1}^{k} w_{ig} d_{ig}^2 \quad (12)$$

where d_{ig} denotes the Euclidean distance between pattern i and center of cluster g. In this study we will also use

this as a distance metric. It has been shown that when the number of clusters exceeds three, the clustering problem is NP-hard [12].

4. Application of Simulated Annealing

4.1 Overview of simulated annealing

The procedure of SA utilizes methods originating from statistical mechanics to find global minima of a given bounded objective function with large degrees of freedom [13].

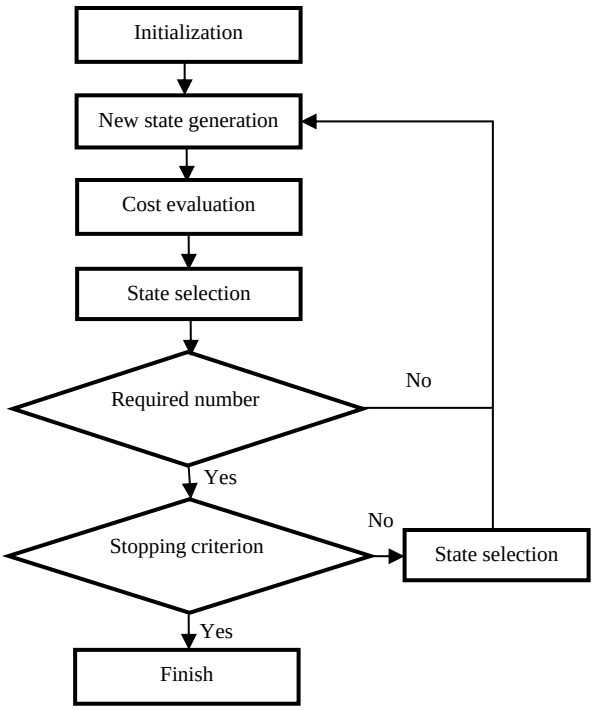

Fig. 1: Flow chart of the SA algorithm

The annealing process can start from any initial state in the domain of interest. According to the selected objective function, the energy of the current state, E_0, is calculated. Then a constraint-based new state is generated from the current one, with energy of E_1. Let ΔE be the energy change of state, $\Delta E = E_1 - E_0$. The next state is decided according to the Metropolis criterion [14]. If the new state is better than the current one ($\Delta E \le 0$), it is accepted unconditionally and becomes the next current state. Otherwise ($\Delta E > 0$), the new state is not rejected outright but accepted with a certain probability. For instance, if $\exp(-\Delta E / T)$ is greater than a random number uniformly distributed in $(0,1)$, the new state is accepted, where T is the control factor 'temperature'. This acceptance of a worse state causes the SA algorithm to escape from local minima. At the beginning the temperature is high to avoid local optima, and thus the probability of acceptance of a worse state remains high. At each T, a series of random new states are generated for selection. Then, an annealing scheme is applied by decreasing T according to some predefined schedule, which lowers the probability of acceptance of a worse state. After some point, a new state is no longer accepted unless it is better than the current one. Fig. 1 outlines the flow chart of the SA algorithm.

4.2 Proposed model using simulated annealing

In this work, a novel model is presented for optimum GPS satellite selection. The SA algorithm is employed in the model to adopt the best answer. The clustering is carried out in the following steps:

Step 1: Set initial parameter values. Let T_i and T_f be the initial and final temperature respectively, m the cooling rate, N the desired Metropolis iteration number, $count$ the counting number of Metropolis iteration and i the counting number of a pattern in the pattern set. The initial values of these parameters are as follows:

$$T_i = 10, T_f = 10^{-99}, m = 0.7 \approx 0.9, N = 4n, count = 0, i = 0$$

Step 2: Randomly assign an initial class label to all of n patterns in k classes and then calculate the objective function J. Let both the optimal objective function value J_b and the current objective function value J_c be J, the corresponding cluster membership matrix of all patterns be W_b. T_b is the temperature corresponding to the optimal objective function J_b. W_c and T_c are the cluster membership matrix and temperature corresponding to the current objective function J_c, respectively. Let $T_c = T_i, T_b = T_i, W_c = W_b$.

Step 3: While the counting number of Metropolis sampling step $count < N$, go to step 4, otherwise, go to step 7.

Step 4: Let $flag = false$ and let p be the probability threshold and if $count \le N/2$, $p = 0.80$, else, $p = 0.95$. A trial assignment matrix W_t can be obtained from the current assignment W_c by the following method.
If $i > n$, then let $i = i - n$, else let $i = i + 1$ take pattern i from the pattern set, initial class assignment (W_{ig}) of this pattern is expressed by f (where f belongs to arbitrary class of k classes), i.e. $f = w_{ig}$. Then draw a

random number u ($u = rand$, where $rand$ is a random number of uniform distribution in the interval [0.1]). If $u > p$, generate a random number r in the range $[1, k]$, here $r \neq f$, put pattern i from class f to class r, let $w_{ig} = r$, let $flag = true$. Otherwise take another pattern, repeat the above process until $flag = true$.

Step 5: Let corresponding trial assignment after above perturbation be W_t. Calculate the objective function value J_t of the assignment. If $J_t \leq J_c$, let $W_c = W_t$, $J_c = J_t$. If $J_t < J_b$, then, $J_b = J_t, W_b = W_t, count = 0$.

Step 6: Produce a random number y. If $y < \exp(-(J_t - J_c)/T_c)$, then $W_c = W_t$, $J_C = J_t$. Otherwise $count = count + 1$, go to step 3.

Step 7: Let $T_c = mT_c$, $count = 0$, $J_c = J_b$, $W_c = W_b$. If $T_c < T_f$ or $T_c / T_b < 10^{-10}$, then stop; otherwise, back to step 3.

5. Results and Discussion

To evaluate the proposed method, computer simulation is performed. The implementation is done in JAVA. The performance of the proposed algorithm depends on its parameters. A preliminary search was performed to obtain the best parameter combination. m is the cooling rate which plays a very important role on the performance of SA algorithm. Since SA has stochastic nature, it is possible to have a different result in each implementation of the algorithm. Therefore the algorithm is executed 10 times for each different value of m in order to find its optimum value and the average result is shown in Fig. 2.

In this work, 937 satellite subsets are used as input to be clustered. The minimum objective function, which means minimum clustering error, is 11.9628 which obtained in cooling rate 0.7.

To measure the quality of anything an index is required. As shown in Table 2, we use purity and entropy as external measures of cluster validity [15]. For each cluster the class distribution of each object is calculated first, i.e. for cluster j we compute p_{ij}, the probability that a member of cluster j belongs to class i as follows: $p_{ij} = m_{ij} / m_j$, where m_j is the number of values in cluster j and m_{ij} is the number of values of class i in cluster j.

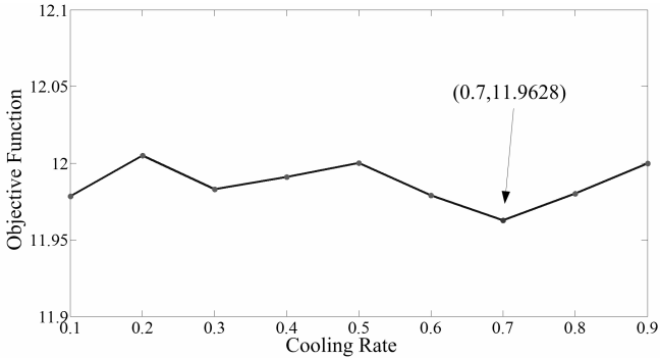

Fig. 2: Objective function according to the cooling rate

Then using this class distribution, the entropy of each cluster j is calculated using the standard formula $e_j = -\sum_{i=1}^{L} p_{ij} \log_2 p_{ij,}$ where the L is the number of classes. The total entropy for a set of clusters is calculated as the weighted sum of the entropies of each cluster, i.e., $e = \sum_{i=1}^{K} \frac{m_j}{m} e_j$, where m_j is the size of cluster j, K is the number of clusters, and m is the total number of data points.

Using the terminology derived for entropy, the purity of cluster j, is given by $purity_j = \max p_{ij}$ and the overall purity of a clustering by $purity = \sum_{i=1}^{K} \frac{m_j}{m} purity_j$. The last two columns in Table 2 show entropy and purity, respectively.

Bad clusterings have purity values close to 0, a perfect clustering has purity of 1. We observe that cluster 2, which contains mainly good GDOP, is a much better (or purer) cluster than the other four. The overall purity is also 0.9015 which is an excellent result. Entropy measures the purity of the clusters with respect to the given class labels. Thus, if every cluster consists of objects with only a single class label, the entropy is 0. Smaller entropy values indicate better clustering solutions. The entropy of cluster 2 is zero and the overall entropy is 0.3993.

Table 2: SA clustering results for GPS GDOP

Cluster	Excellent	Good	Moderate	Fair	Poor	Entropy	Purity
1	206	2	0	0	0	0.0782	0.9903
2	0	134	0	0	0	0	1
3	0	12	120	0	0	0.4395	0.9090
4	0	0	34	122	0	0.7563	0.7820
5	0	0	0	44	263	0.5929	0.8566
Total	206	136	154	166	263	0.3993	0.9015

6. Conclusion

This paper proposes a novel method for optimum GPS satellite selection. A SA clustering algorithm is employed to minimize the objective function in order to obtain the best subset whose geometry is most similar to optimal subset among all the visible satellites. The disadvantage of NN-based algorithms is that they require training in advance. So the performance of such algorithms is affected by the location that training data are collected and the position of receivers. But this is not the case for the proposed method because the there is no training step. SA clustering also reduces the computational burden because it does not need to use matrix inversion for classifying GPS satellite subsets.

Two indices are used for evaluating the method, purity and entropy. The greater the value of purity indicates good clustering. The entropy is negative measure, the lower the entropy the better clustering it is. The experimental results show that the SA clustering for GPS subset selection has high value of purity and low value of entropy. This indicates good clustering.

References

[1] A. EI-Rabbany, "Introduction to GPS: The Global Positioning System", Boston, MA: Artech House, 2002.

[2] R. Yarlagadda, I. Ali, N. Al-Dhahir and J. Hershey, "GPS GDOP Metric", IEEE Proceedings on Radar, Sonar and Navigation, Vol.147, pp.259-264, 2000.

[3] D. Y. Hsu, "Relations between Dilutions of Precision and Volume of the Tetrahedron Formed by Four Satellites", IEEE Symposium on Position, Location and Navigation, pp.669-676, 1994.

[4] D. Simon and H. El-Sherief, "Navigation Satellite Selection Using Neural Networks", Journal of Neurocomputing, Vol.7, pp.247-258, 1995.

[5] M. Saraf, K. Mohammadi and M. R. Mosavi, "Bayesian Framework on GPS GDOP Classification", Journal of Computers & Electrical Engineering, Vol.37, pp.1009-1018, 2011.

[6] M. R. Mosavi and H. Azami, "Applying Neural Network for Clustering of GPS Satellites", Journal of Geoinformatics, Vol.7, No.3, pp.7-14, 2011.

[7] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by Simulated Annealing", Journal of Science, Vol.220, pp.671-680, 1983.

[8] I. O. Bohachevsky, M. E. Johnson, M. L. Stein and I. Bohachevsky, "Generalized Simulated Annealing for Function Optimization", Journal of Technometrics, Vol.28, pp.209-217, 1986.

[9] P. Laarhoven and E. Aarts, "Simulated Annealing: Theory and Applications", Journal of Math. Its Appl., Vol.12, pp.108-111.

[10] S. H. Doong, "A Closed-Form Formula for GPS GDOP Computation", Journal of GPS Solutions, Vol.13, pp.183-190, 2008.

[11] Z. Güngör and A. Ünler, "K-harmonic Means Data Clustering with Simulated Annealing Heuristic", Journal of Applied Mathematics and Computation, Vol.184, pp.199-209, 2007.

[12] P. Brucker, "On the Complexity of Clustering Problems", In M. Beckmenn & H. P. Kunzi (Eds.), Optimisation and Operations Research, Lecture Notes in Economics and Mathematical Systems, Vol.157, pp.45-54, 1978.

[13] A. Y. Zomaya, "Natural and Simulated Annealing", Journal of Computing in Science & Engineering, Vol.3, pp.97-99, 2001.

[14] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller and E. Teller, "Equations of State Calculations by Fast Computing Machines", Journal of Chem. Phys., Vol.2, pp.1087-1982, 1953.

[15] Y. Zhao and G. Karypis, "Criterion Functions for Document Clustering: Experiments and Analysis", Technical Report TR 01-40, Department of Computer Science, University of Minnesota, 2001.

Mostafa Ranjbar received his B.S. and M.S. degrees in computer engineering from Iran University of Science and Technology, Tehran, Iran in 2010 and 2012 respectively. His research interests include Global Positioning Systems, data mining, and video processing.

Mohammad-Reza Mosavi (Corresponding Author) received his B.S., M.S. and Ph.D. degrees in Electronic Engineering from Iran University of Science and Technology (IUST), Tehran, Iran in 1997, 1998 and 2004, respectively. He is currently faculty member of Department of Electrical Engineering of IUST as associate professor. He is the author of about 150 scientific publications on journals and international conferences. His research interests include circuits and systems design.