Wave Propagation in Rectangular Waveguide Filled with Anisotropic Metamaterial

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Abstract

In this paper a rigorous analysis of the Transverse Operator Method (TOM) followed by the application of the Galerkin method, is developed for studying the propagation in a rectangular metal waveguide filled with anisotropic metamaterial. The wave equation and the dispersion relations for guided and evanescent modes in the guide are obtained and analyzed. The higher order modes are exploited. The numerical results are obtained and compared to theoretical predictions. Numerical examples show the validity of this method. Anisotropic metamaterials found important industrial applications such as circulators, isolators, phase shifters and antennas.

Keywords: *Galerkin method, Higher order modes, Rectangular waveguide with anisotropic metamaterial, Transverse operator method*.

1. Introduction

Double negative materials (DNG) with negative permittivity and permeability, enjoy a growing interest, especially because of their physical properties that are different from those of conventional double positive materials (DPS). Veselago [1] was the first to study theoretically the DNG materials. Different aspects of this class of metamaterials have been studied eg in [2-3]. Interesting features of the guided modes in waveguide filled with two parallel planes materials DPS and DNG have been studied in [4-8]. [9] and [10] have studied the propagation in waveguides to metamaterials.

In this paper, we present an analysis of the propagation in rectangular metal waveguides completions filled with anisotropic metamaterial (Fig. 1) using the TOM followed by the Galerkin method. Numerical examples are given by exploiting the higher order modes in these types of structure. The results are compared to those of conventional double positive materials (DPS).

MOT [11] takes into account the spatial distribution of the permittivity and permeability of media and therefore, discontinuities of the latter which is applied to the transversal field. This method is applicable to various structures, homogeneous or inhomogeneous isotropic or anisotropic. The convergence of constant propagation of the studied structure is fast. It is obtained for N = 15 modes.

2. Analysis

2.1 The transverse operator method

In a rectangular coordinate system, we consider a metallic rectangular waveguide of width a and high b as shown in figure 1, comprising an anisotropic medium characterized by a permittivity and relative permeability tensor given by the equation (1) and (2).

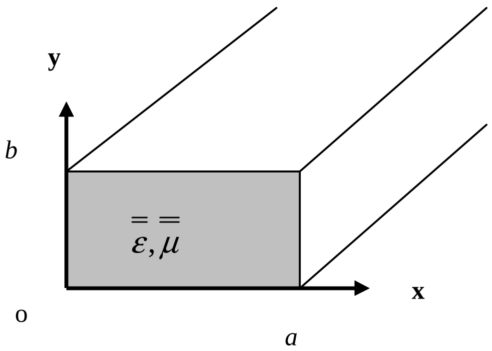


Fig. 1 Configuration of the rectangular waveguide completely filled with anisotropic metamaterial

$$\bar{\bar{\varepsilon}}_r = \begin{bmatrix} \bar{\bar{\varepsilon}}_t & \varepsilon_{tz} \\ \varepsilon_{zt} & \varepsilon_{zz} \end{bmatrix} \; ; \; \bar{\bar{\mu}}_r = \begin{bmatrix} \bar{\bar{\mu}}_t & \mu_{tz} \\ \mu_{zt} & \mu_{zz} \end{bmatrix} \qquad (1)$$

with :

$$\bar{\bar{\varepsilon}}_t = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}, \; \varepsilon_{tz} = \begin{bmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{bmatrix}, \; \varepsilon_{zt} = \begin{bmatrix} \varepsilon_{zx} & \varepsilon_{zy} \end{bmatrix}$$

$$\overline{\overline{\mu}}_t = \begin{bmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{bmatrix}, \mu_{tz} = \begin{bmatrix} \mu_{xz} \\ \mu_{yz} \end{bmatrix}, \mu_{zt} = \begin{bmatrix} \mu_{zx} & \mu_{zy} \end{bmatrix} \quad (2)$$

Maxwell's equations are written:

$$rot\,\vec{E} = -j\omega\mu_0 \overline{\overline{\mu}}_r . \vec{H} \quad (3)$$

$$rot\,\vec{H} = j\omega\varepsilon_0 \overline{\overline{\varepsilon}}_r . \vec{E} \quad (4)$$

Considering a propagation along Oz, we have:

$$\Phi(x,y,z) = \Phi_t(x,y).\exp(-jk_z z) \quad (5)$$

with

$$\Phi_t = [E_t \quad H_t^{'}]^t,\ H^{'} = j\sqrt{\mu_0/\varepsilon_0}.H \quad (6)$$

Φ_t : Represents the transverse components of the electromagnetic fields, ω is the angular frequency, k_0 and Z_0 are respectively; the propagation constant and the characteristic impedance of free space.

By eliminating the longitudinal components of electromagnetic fields, (3) and (4) can be written [11]:

$$\hat{L}\Phi_t = j\eta\partial_z\Phi_t \quad (7)$$

$\hat{L}$ is the transverse operator defined by:

$$\hat{L} = \begin{bmatrix} \hat{L}_{11} & \hat{L}_{12} \\ \hat{L}_{21} & \hat{L}_{22} \end{bmatrix} \quad (8)$$

with :

$$\hat{L}_{11} = k_0 \overline{\overline{\varepsilon}}_E - 1/k_0\partial_t[1/\mu_{zz}\partial_t^+] \quad (9)$$

$$\hat{L}_{12} = -\varepsilon_{tz}/\varepsilon_{zz}\partial_t^+ - \partial_t\mu_{zt}/\mu_{zz} \quad (10)$$

$$\hat{L}_{21} = -\mu_{tz}/\mu_{zz}\partial_t^+ - \partial_t\varepsilon_{zt}/\varepsilon_{zz} \quad (11)$$

$$\hat{L}_{22} = k_0 \overline{\overline{\mu}}_E - 1/k_0\partial_t[1/\varepsilon_{zz}\partial_t^+] \quad (12)$$

We have:

$$\overline{\overline{\varepsilon}}_E = \overline{\overline{\varepsilon}}_t - \varepsilon_{tz}\varepsilon_{zt}/\varepsilon_{zz}\ ,\ \partial_t^+ = [-\partial_y \quad \partial_x]\ ,$$

$$\overline{\overline{\mu}}_E = \overline{\overline{\mu}}_t - \mu_{tz}\mu_{zt}/\mu_{zz}\ ,\ k_0 = \omega\sqrt{\varepsilon_0\mu_0}\ ,$$

$$\partial_t = \begin{bmatrix} \partial_y \\ -\partial_x \end{bmatrix}, \eta = \begin{bmatrix} 0 & \eta_0 \\ \eta_0 & 0 \end{bmatrix}, \text{and } \eta_0 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \quad (13)$$

∂_t is the transversal operator and ∂_t^+ is its adjoin operator.

The longitudinal components are related to the transverse fields by:

$$H_z^{'} = -\mu_{zt}H_t^{'}/\mu_{zz} + \partial_t^+ E_t/\mu_{zz}/k_0 \quad (14)$$

$$E_z = -\varepsilon_{zt}E_t/\varepsilon_{zz} + \partial_t^+ H_t^{'}/\varepsilon_{zz}/k_0 \quad (15)$$

The above mentioned relations are valid for solving problems in inhomogeneous media guide with, isotropic or anisotropic and dissipative.

Analyzing the case of anisotropic metamaterials diagonals such as:

$$\overline{\overline{\mu}}_r = \begin{bmatrix} \mu_{rx} & 0 & 0 \\ 0 & \mu_{ry} & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} \quad (16)$$

$$\text{and } \overline{\overline{\varepsilon}}_r = \begin{bmatrix} \varepsilon_{rx} & 0 & 0 \\ 0 & \varepsilon_{ry} & 0 \\ 0 & 0 & \varepsilon_{rz} \end{bmatrix} \quad (17)$$

These conditions lead to that $\hat{L}_{12} = \hat{L}_{21} = 0$. As a result the equation (7) becomes

$$\partial_z E_t = -j\eta_0\hat{L}_{22}H_t^{'} \quad (18)$$

$$\partial_z H_t^{'} = -j\eta_0\hat{L}_{11}E_t \quad (19)$$

That $\hat{L}$ is independent of z, by deriving the system (18) and (19) with respect to z, we obtain two decoupled equations in E_t and $H_t^{'}$.

$$\hat{L}^{'}\Phi_t = k_z^2\eta^{'}\Phi_t \quad (20)$$

with

$$\hat{L}^{'} = \begin{bmatrix} \hat{L}^{'}_{11} & 0 \\ 0 & \hat{L}^{'}_{22} \end{bmatrix} \text{ and } \eta^{'} = \begin{bmatrix} \eta_0 & 0 \\ 0 & \eta_0 \end{bmatrix}. \quad (21)$$

In these expressions:

$$\hat{L}^{'}_{11} = \hat{L}_{22}\,\eta_0\,\hat{L}_{11}\ , \quad (22)$$

$$\hat{L}^{'}_{22} = \hat{L}_{11}\,\eta_0\,\hat{L}_{22} \quad (23)$$

The decoupled equation with $\vec{E}_t$ becomes:

$$\hat{L}^{''} E_t = k_z^2 E_t \quad (24)$$

$$\text{Or: } \begin{cases} \hat{L}^{''}_{11}E_x + \hat{L}^{''}_{12}E_y = k_z^2 E_x \\ \hat{L}^{''}_{21}E_x + \hat{L}^{''}_{22}E_y = k_z^2 E_y \end{cases} \quad (25)$$

with

$$\hat{L}^{''}_{11} = k_0^2\varepsilon_{rx}\mu_{ry} + \frac{\varepsilon_{rx}}{\varepsilon_{rz}}\partial_x^2 + \frac{\mu_{ry}}{\mu_{rz}}\partial_y^2 \quad (26)$$

$$\hat{L}^{''}_{12} = \frac{\varepsilon_{ry}}{\varepsilon_{rz}}\partial_x\partial_y - \frac{\mu_{ry}}{\mu_{rz}}\partial_y\partial_x \quad (27)$$

$$\hat{L}^{''}_{21} = \frac{\varepsilon_{rx}}{\varepsilon_{rz}}\partial_y\partial_x - \frac{\mu_{rx}}{\mu_{rz}}\partial_x\partial_y \quad (28)$$

$$\hat{L}''_{22} = k_0^2 \varepsilon_{ry} \mu_{rx} + \frac{\mu_{rx}}{\mu_{rz}} \partial_x^2 + \frac{\varepsilon_{ry}}{\varepsilon_{rz}} \partial_y^2 \tag{29}$$

Equation (24) is an eigenvalue equation.

The decomposition of the fields $\vec{E}_T$ on a complete system provides an eigenvalue. The terms of transverse fields, satisfying the boundary conditions ($E_T = 0$, for $x = 0$ or a, $y = 0$ or b) can be written in the following forms

$$E_x = \sum_{m,n=0}^{N} E_{x,mn} \cos\frac{m\pi}{a} x . \sin\frac{n\pi}{b} y = \sum_{m,n=0}^{N} E_x^{mn} \tag{30}$$

$$E_y = \sum_{m,n=0}^{N} E_{y,mn} \sin\frac{m\pi}{a} x . \cos\frac{n\pi}{b} y = \sum_{m,n=0}^{N} E_y^{mn} \tag{31}$$

We note:

$$f_m^x = \sin(\frac{m\pi}{a} x), \qquad g_m^x = \cos(\frac{m\pi}{a} x)$$

$$f_n^y = \sin(\frac{n\pi}{b} y), \qquad g_n^y = \cos(\frac{n\pi}{b} y), \tag{32}$$

The system of equations (25) can be written

$$\sum_{mn} \hat{L}''_{11} E_{x,mn} g_m^x f_n^y + \hat{L}''_{12} E_{y,mn} f_m^x g_n^y = k_z^2 \sum_{mn} E_{x,mn} f_n^y g_m^x \tag{33}$$

$$\sum_{mn} \hat{L}''_{21} E_{x,mn} g_m^x f_n^y + \hat{L}''_{22} E_{y,mn} f_m^x g_n^y = k_z^2 \sum_{mn} E_{y,mn} f_m^x g_n^y \tag{34}$$

2.2 Application of the Galerkin method

We have: $0 \le x \le a \;\; ; 0 \le y \le b$.

One can choose the following test functions

$$f_{m'}^x = \sin(\frac{m'\pi}{a} x), \quad g_{m'}^x = \cos(\frac{m'\pi}{a} x)$$

$$f_{n'}^y = \sin(\frac{n'\pi}{b} y), \quad g_{n'}^y = \cos(\frac{n'\pi}{b} y) \tag{35}$$

The inner products $f_{n'}^y g_{m'}^x$ with equation (33) and $f_{m'}^x g_{n'}^y$ with equation (34), then integrating, we obtain

$$\sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} [\hat{L}''_{11} E_{x,mn} g_{m'}^x g_m^x f_{n'}^y f_n^y + \hat{L}''_{12} E_{y,mn} g_{m'}^x f_m^x f_{n'}^y g_n^y]\, dxdy = k^2 \sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} E_{x,mn} g_{m'}^x g_m^x f_{n'}^y f_n^y .dxdy \tag{36}$$

$$\sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} [\hat{L}''_{21} E_{x,mn} f_{m'}^x g_m^x g_{n'}^y f_n^y + \hat{L}''_{22} E_{y,mn} f_{m'}^x f_m^x g_{n'}^y g_n^y]\, dxdy = k_z^2 \sum_{m',n',m,n} \int_{(O,O)}^{(a,b)} E_{y,mn} g_{n'}^y g_n^y f_{m'}^x f_m^x .dxdy \tag{37}$$

they can be written in the following matrix form

$$\sum_{m',n',m,n}^{N} \begin{bmatrix} H_{11\,mn}^{m'n'} & H_{12\,mn}^{m'n'} \\ H_{21\,mn}^{m'n'} & H_{22\,mn}^{m'n'} \end{bmatrix} \cdot \begin{bmatrix} E_{x,mn} \\ E_{y,mn} \end{bmatrix} = k_z^2 \sum_{m',n',m,n}^{N} \begin{bmatrix} G_{11\,mn}^{m'n'} & 0 \\ 0 & G_{22\,mn}^{m'n'} \end{bmatrix} \cdot \begin{bmatrix} E_{x,mn} \\ E_{y,mn} \end{bmatrix} \tag{38}$$

By normalising $f_m^x, f_n^y, g_m^x, g_n^y$, we can easily find that: $G_{11\,mn}^{m'n'} = G_{22\,mn}^{m'n'} = 1$.

We set: $$\mathrm{H} = \begin{bmatrix} H_{11\,mn}^{m'n'} & H_{12\,mn}^{m'n'} \\ H_{21\,mn}^{m'n'} & H_{22\,mn}^{m'n'} \end{bmatrix}. \tag{39}$$

The system (38) can be written

$$H \cdot E_T = k^2 \cdot I \cdot E_T \tag{40}$$

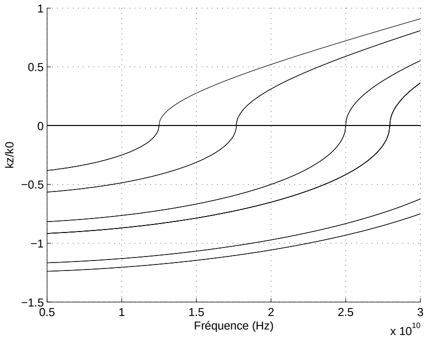
I is the identity matrix. H is a square matrix of order $2(N-1)$ with N = mn: number of modes; m and n are natural numbers, such as: $(m,n) \ne (0,0)$.

The eigenvalues and the proper vectors of H are respectively the propagation constant and the coefficients of development of the field of the guide.

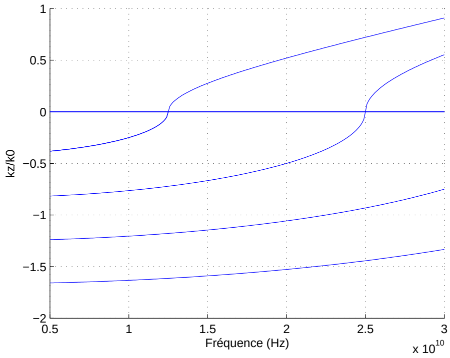
3. Simulation Results

3.1 Waveguide No. 1

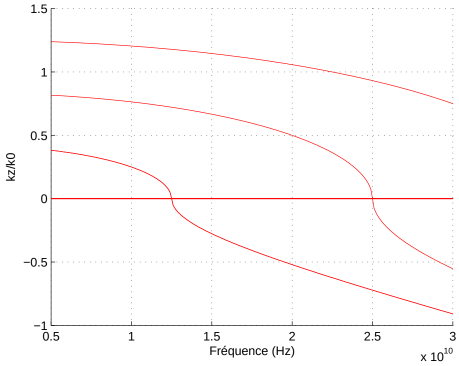
Consider No. 1 a metal square waveguide: side a = b = 12 mm [2] completely filled with metamaterial.



(a)



(b)

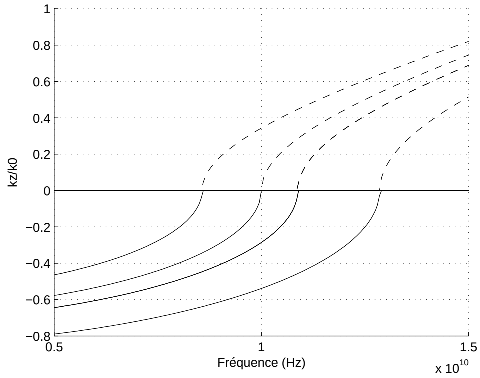


(c)

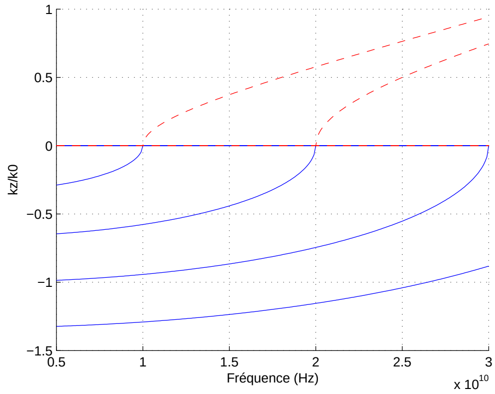
Fig. 2 Dispersion curves of the guide No. 1:
(a) empty guide;
(b) guide filled with isotropic metamaterial where,
$\varepsilon_{rx} = \varepsilon_{ry} = \varepsilon_{rz} = -1, \mu_{rx} = \mu_{ry} = \mu_{rz} = -1,$
(c) guide filled with anisotropic metamaterial where
$\varepsilon_{rx} = \varepsilon_{ry} = \varepsilon_{rz} = -1,\ \mu_{rx} = \mu_{ry} = 1,\ \mu_{rz} = -1.$

3.2 Waveguide No. 2

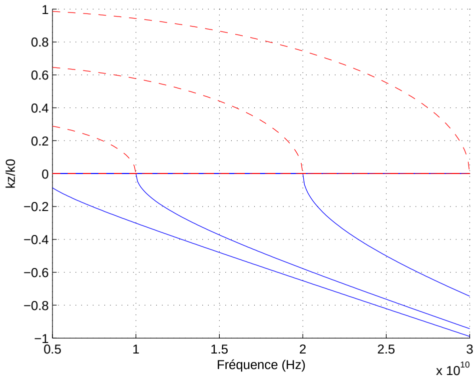
Consider No. 2 a metal rectangular waveguide: a = 35 mm and b = 15mm [2] completely filled with metamaterial.



(a)



(b)



(c)

Fig. 3 Dispersion curves Guide No.2:
(a) empty guide,
(b) guide filled with isotropic metamaterial where
$\varepsilon_{rx} = \varepsilon_{ry} = \varepsilon_{rz} = -1, \mu_{rx} = \mu_{ry} = \mu_{rz} = -1,$
(c) guide filled with anisotropic metamaterial where
$\varepsilon_{rx} = \varepsilon_{ry} = \varepsilon_{rz} = -1,\ \mu_{rx} = \mu_{ry} = 1,\ \mu_{rz} = -1.$

On the one hand we notice that there is a spread in the guides completely filled with metamaterial (DNG) isotropic. On the other hand, the cutoff frequencies of isotropic metamaterial guides change over the same guide vacuum.

Among the particularities of this anisotropic material, the backward and forward waves can both propagate below the cutoff frequency in the guide. The numerical results obtained in this paper are agree with the references [2] and [5] which validates our numerical calculations.

4. Conclusions

With the formalism of the TOM, we presented a rigorous study of propagation in homogeneous anisotropic media using the tensor character of the permeability and permittivity. With the application of the Galerkin method we have studied the evanescent and propagating modes in rectangular guides filled with isotropic or anisotropic metamaterial. Comparisons to guides containing a conventional dielectric are exploited.

The advantages of the techniques used in this paper lies in the proper analytical formulation of the problem studied on the one hand and the speed of convergence on the other. This type of materials known as metamaterial is widely used and needed by industries and information technology, especially in microwave and RF devices such as patch antennas, the antennas waveguides, resonators, circulators, isolators, phase shifters ...

TOM offers a fast convergence of the propagation constant. This shows the effectiveness of our numerical model. As such, the formulation of the transverse operator could be a useful tool for microwave engineers.

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