Modelling of gastrocnemius muscle using Hill's equation in COMSOL Multiphysics 4.0a

S.Vivekanandan[1], D.S.Emmanuel[2] and Ramandeep Singh Saluja[3]

**[1,3]School of Electrical Engineering, VIT University
Vellore, Tamil Nadu- 632014, India**

**[2]School of Electronics Engineering, VIT University
Vellore, Tamil Nadu- 632014, India**

Abstract

This paper summarizes the force generated by gastrocnemius muscle for the analysis of muscoskeletal simulation in human locomotion using Hill's muscle model. Biomechanics of Hill's equation describes the study of physical phenomenon by means of mathematical model that relates force and muscle length with the help of a partial differential equation. To calculate maximum fatigue in the muscle and to discriminate strained muscle from the normal one FEM based modelling was done in COMSOL Multiphysics 4.0a. The model parameters were evaluated using similar *in vitro* experiments performed on frog's gastrocnemius muscle. The biomechanical model was then incorporated into human body for the purpose of predicting force - length response for all the four phases of gait cycle. Evaluating the response for gait cycle will enable the physiotherapist to obtain clues for muscle weakness and fatigue in a rehabilitation program and will lead to more efficient gait pattern resulting in decreased risk of injury and improved muscular balance.

Keywords: *Biomechanics, Hill's equation, gastrocnemius muscle, gait analysis, COMSOL Multiphysics 4.0a.*

1. Introduction

The attractiveness of modeling lies in addressing research related question to test several types of models. Mathematical model are used to capture and explore a wide range of real world settings. Hill's muscle model is a mathematical abstraction or simplification of reality. There are two different types of models that are typically used to simulate the biomechanics of a muscle. The first of these is Huxley and Simmon which is well adapted to account for the cross bridge mechanics as already reported [1] [5]. The second is Hill's muscle model, also known as three-element model for tetanized muscle contraction that provides the analogy between force and muscle length in an intrafusal fibres.

The mechanism of muscle contraction is provoked by the impulse from the spinal cord. As the impulse travels from neuron to sarcolemma, sarcoplasmic reticulum releases calcium ions. These calcium ions bind to troponin molecules and alter their shape. This causes tropmyosin molecules to move which results in myosin head to swivel and contact actin molecules as shown in Fig 1. During the swivel, the myosin head is firmly attached to actin which pulls it forward and results in muscle contraction [3] [13].

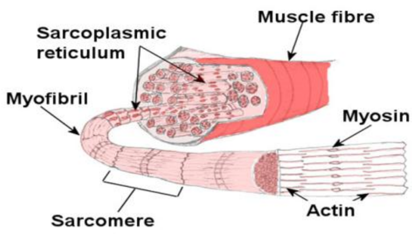


Fig1.Muscle fibre

Assessment of muscular contraction during locomotion provides insight in to the biomechanics of human gastrocnemius muscle that is essential in gait analysis [6]. For an athlete during running, walking or any vigorous exercise the force generated by the muscle is a direct function of elongation or contraction produced in it [7]. Hill's equation describes the interdependence of force and lengthening produced in the muscle. Implementing this as a general form partial differential equation (PDE) in COMSOL gives us an inside view of the force generated in each segment of the muscle and its variation from origin to the surface of the gastrocnemius muscle.

2. Hill's muscle model

Muscles produce more force when they are stretched than during contraction. A. V. Hill derived an empirical relation that describes the relationship between force and length of a muscle during a brief tetanic isometric contraction in the year 1922. His work quantifies the total force produced in the muscle which is a combination of both active and passive components [2]. For this, he developed biomechanical model of the muscle that predicts the force– length response for both fast and slow twitch muscles in mathematical domain as shown in Fig 2.

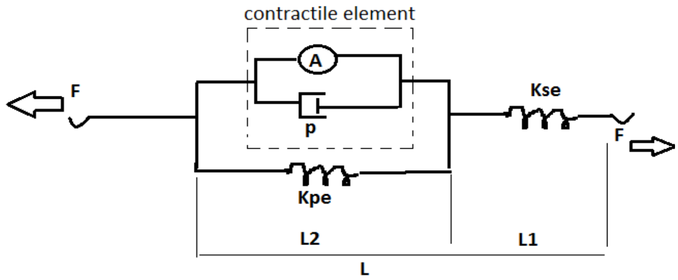


Fig 2. Mechanical model of muscle

The above model describes muscle mechanics by combination of contractile element, series elastic element (SEE) and parallel elastic element (PEE). Contractile element consists of 2 components - a force generator and a dashpot. The dashpot which has a damping constant provides for the viscous force of a muscle and is a nonlinear function of shortening velocity and temperature. The function of the contractile element is to convert nervous signal from neurons into a force. The tendon attached to the motor unit constitutes the SEE which acts as a spring and accommodates force deflection properties of a tendon. The PEE describes passive or resistive nature of the muscle and incorporates force deflection properties of sarcolemma, epimysium, perimysium and endomysium [4] [14].

Hill's equation is based on the assumptions that for a given length, muscle always develops the same peak force and if the muscle is shortening some force is dissipated in overcoming inherent viscous force. The Hill's equation relates the two time dependent variables- force and length in the intrafusal fibres of gastrocnemius muscle [7]. To derive Hill's equation, the basic equation (3.1) for a spring that relates force and length with a constant of proportionality 'K' as spring's stiffness constant was used. Similarly applying the equation on series elastic component with stiffness constant as K_{se} and on parallel elastic component with stiffness constant as K_{pe} we obtain equation (3.2) and equation (3.3) respectively [2].

$$K = \frac{\Delta F}{\Delta L} \quad (3.1)$$

$$F = K_{se}(L_1 - L_1^*) \quad (3.2)$$

and

$$F = K_{pe}(L_2 - L_2^*) + p \cdot \frac{\partial L_2}{\partial t} + A \quad (3.3)$$

Where L_1 and L_2 are the original length and L_1* and L_2* are the resting length of SEE and PEE respectively, p is the damping constant and A is the active force component of contractile element.

Summing up the length L_1 and L_2 of SEE and PEE for muscles total length and differentiating with respect to time (t) gives the partial differential equation (3.4) which shows the relation between rate of tension produced in the intrafusal fibres as a function of length.

$$\frac{\partial F}{\partial t} = \frac{K_{se}}{p}\left(K_{pe} \cdot \Delta L + p \cdot \frac{\partial L}{\partial t} - \left(1 + \frac{K_{pe}}{K_{se}}\right)F + A\right) \quad (3.4)$$

To calculate value of parameters such as K_{se}, K_{pe} and p, tension in frog gastrocnemius calf muscle was observed for various conditions [2]. Frog muscle was used because it is easy to work with isolated muscle that can be tested analytically in laboratory than to work with complex machine like that of human being [8].

When the muscle is stretched PEE cannot move quickly because of its damping, so most of the length changes occur in SEE of spring. We can measure the change in tension for a given change in length. This allows us to calculate the K_{se} spring constant as shown below in equation (3.5).

$$K_{se} = \frac{\Delta F}{\Delta L} = \frac{150}{1.1} = 136\ g/cm \quad (3.5)$$

There will be an increase in tension when muscle reaches the steady state after the stretch. This is due to the passive tension of the PEE of the muscle. We can use this to estimate K_{pe} as shown below in equation (3.6).

$$K_{pe} = \frac{K_{se}}{\left(\frac{\Delta L}{\Delta F} \cdot K_{se} - 1\right)} \approx 75 g/cm \quad (3.6)$$

After being stretched for some time, muscle force declines to a new steady state. We can use this to calculate time constant of decay and damping coefficient with help of equation (3.7). For frog's calf muscle $\tau \approx 0.23$ as reported in [2].

$$p = \tau \cdot (K_{se} + K_{pe}) = 0.23(136+75) \approx 50 g \cdot s/cm \quad (3.7)$$

Similarly another set of values are determined from the experimental data and applied on this model to study force-length responses for gastrocnemius muscle.

3. Implementation in COMSOL Multiphysics 4.0a

Theoretical models can be easily customized in COMSOL to solve any prototype for a predefined physics module. To simulate Hill's muscle equation using COMSOL we need to specify the geometry, type of physics involved, material properties, source terms that are all the arbitrary functions of the dependent variable [12].

4.1 Geometry

To simulate muscle, cylindrical geometry with three dimensional form of COMSOL's PDE is used. There is large number of internal cylinders which denotes intrafusal fibre of the gastrocnemius muscle. The cylindrical segment with a length of 1cm and area of 12.56 cm^2 is used for modeling equation (3.4) as shown in Fig 3.

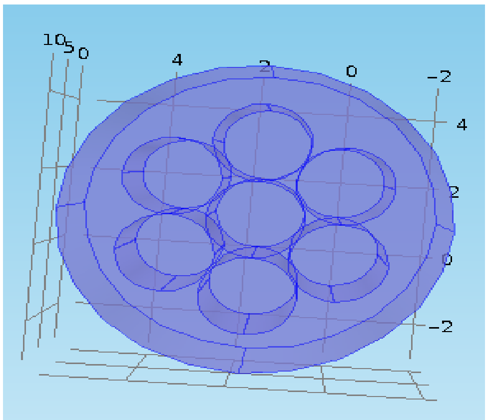


Fig.3 Geometry of muscle

4.2 Type of Module and material properties involved

PDE module is used to implement the Hill's equation and solid mechanics module is used to specify the viscoelastic properties of the muscle [11]. The various parameters that need to be specified for muscle are listed in the table (4.1).

Table (4.1) Material properties

Name	Value	Description
E	3.48 N/cm^2	Young's modulus
V	.28	Poisson's ratio
G	.00026 GPa	Long-term shear modulus
K	.000135 Pa	Bulk-Modulus

In PDE module, two dependent variables 'F' and 'L' are selected for implementing Hill's equation. The general mode PDE solves the following equation (4.1)

$$\frac{e_a \partial^2 u}{\partial t^2} + \frac{d_a \partial u}{\partial t} + \nabla \cdot \Gamma = f \qquad (4.1)$$

Where u = $[F\ L]^T$ and e_a is the mass coefficient, d_a is damping coefficient, Γ is conservative flux and f is the source term. To build the Hill's model parameters are set as given in global definitions [10].

4.3 Global definition

All the parameters related to Hill's equation such as K_{se}, K_{pe} and p are calculated using equations (3.5), (3.6) and (3.7) respectively and tabulated in (4.2).

Table 4.2 Parameters for Hill's Equation

K_{se}	K_{pe}	*P*	A_1 *(force vs time)*	A_2 *(length vs time)*	A_3 *(force vs length)*
136	75	50	200	12.19	-86
90.9	-30.3	13.93	50	55.79	30.03
45.45	45.4509	20.909	174	15.51	174
45.45	-90.89	-10.45	100	-70	98.5
72.72	24.24	22.3	130	15.01	130

4.4 Boundary conditions

There are two types of boundary conditions one is Neumann boundary condition given by equation (4.2) and the other one is Drichlet boundary condition given by equation (4.3).

$$-n \cdot \Gamma = G + \left(\frac{\partial R}{\partial u}\right)^T \mu \qquad (4.2)$$

$$R=0 \qquad (4.3)$$

Where the term Γ is flux vector, G and R are scalars and functions of spatial coordinates of u. T in Neumann boundary condition denotes transpose and μ is Lagrange multiplier[9] [12].

4.5 Meshing of geometry and study settings

A finite element approach approximates PDE into a problem that has a finite number of unknown parameters *i.e.*, a discretization of the original problem. Thus the finite elements describe the possible forms of the approximate solution. There are various options available to mesh the entire geometry into finite elements but due to memory constraint we have modelled this as a tetrahedral mesh with each element size as coarser one. As this model deals with the time dependent equation we have set its range from 0 to 1 in steps of 0.1. MUMPS solver is used in this model.

5. Result

To determine the model's effectiveness under varying load, Hill's equation was analysed for different value of parameters as listed in Table (4.2). For the gastrocnemius muscles of frog, the model accurately fitted and predicted the force responses of human gastrocnemius muscle as a function of length as already reported in [8]. Fig. 4 shows the simulation for force with respect to length as a function of time in COMSOL Multiphysics for different values of load, assuming that muscle is at rest at $t = 0$ and its resting length is 1 cm. A brief twitch of 0.05 s causes muscle to increase its length by 1 cm from resting length. This also causes the force to increase linearly from initial value to a maximum value of 139.1 g as shown in Fig 5. This is the point of maximum fatigue after which muscle length does not lengthen. Force then parabolically decreases during which muscle length does not change. After .95 s of initial twitch muscle comes back to its resting length. Since skeletal muscle produce maximum force when they contract from resting length, a maximum force of 139.1 g is produced at the centre, and since force is directly proportional to the lengthening produced so this force corresponds to an elongation of 2.12 cm which decreases as it reaches the surface.

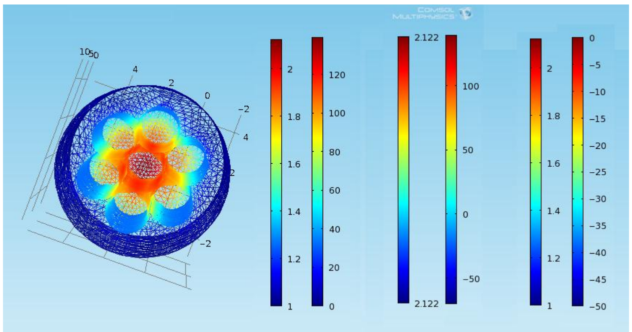
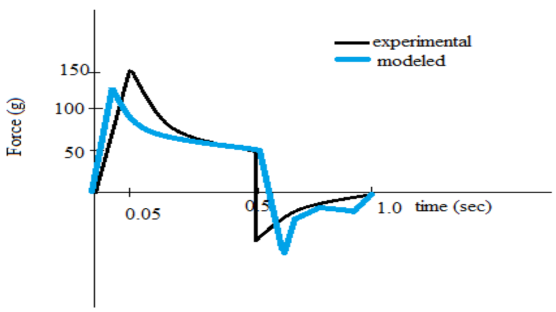
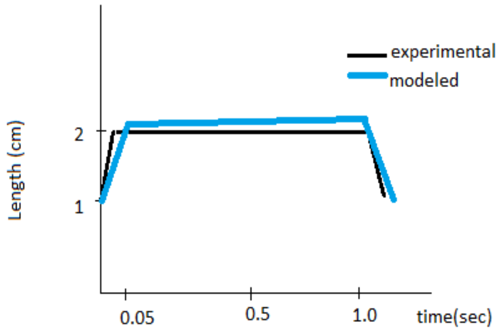


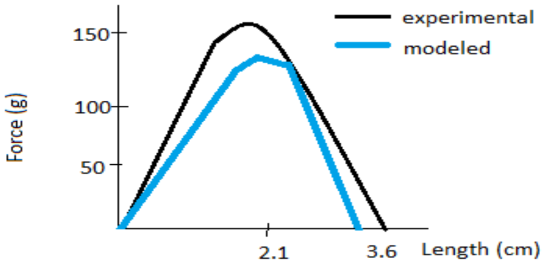
Fig.4 Variation of force and length in the muscle as a function of time.



(a)



(b)



(c)

Fig.5 Response for (a) force vs time (b)Length vs time (c) Force vs Length.

6. Conclusion

This model accurately predicts the force generated by human gastrocnemius muscle for all 4 phases of gait. To evaluate the validity of the model, several comparisons were made between the modelled and experimental data. Implementation of force, length and time integral in COMSOL Multiphysics showed that maximum force generated at the centre was 139.1 N which corresponds to an elongation of 2.12 cm. so we observe that maximum fatigue occurs at resting length. At length less than 70% of resting length muscle produces no tension. Maximum isometric contraction of the muscle is obtained when the muscle is stretched to about 180-200% of its resting length and length is held constant at this value till the contraction is induced. This is the maximum stretch without any damage to muscle. Among the model parameters series elastic component responds immediately when the tension in the muscle suddenly decreases while parallel elastic component responds gradually because of viscous component. Although this model accurately predicts the isometric force for athletes in gait analysis, it failed to explain the negative force at the end of twitch and to predict the force at the time of lengthening.

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S.Vivekanandan is currently an Assistant Professor (senior) .He is doing his PhD degree and completed his M.Tech from Annamalai University in the year of 2005 .His Research area is biomedical modeling, signal processing and data mining. He is also faculty adviser of ISA and has written various Research paper in various journals.

Dr. DS Emmanuel took his BE, MSc (Eng.) and PhD degrees from the Madurai, Madras and Roorkee Universities in the year 1973, 1975 and 1989 respectively. He has been teaching for 37 years. He won the Best Teacher award of the Rotary Club, Allahabad in 1993.He joined VIT in 2003 and is now a senior Professor and Program manager, BTech. (ECE).His research interest is in Biomedical Instrumentation. He has PhD scholars working under him in a broad spectrum covering biomedical signals, data fusion, microstrip antenna design, spectrum sensing, etc. He has published/presented papers in national/International journals/conferences. He is a member of IEEE and ISTE.

Ramandeep Singh Saluja is Final year student doing Btech. in Electronics and Instrumentation from VIT University. His research interest includes Biomedical Instrumentation and Signal Processing. He is active member of ISA and ISOI.