IJCSI International Journal of Computer Science Issues, Vol. 9, Issue 4, No 1, July 2012
ISSN (Online): 1694-0814
www.IJCSI.org

A Hybrid Model for Estimation of Volatility of Call Option Price Using Particle Filter

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Abstract

In the recent years, the distribution of possible future losses for portfolios, such as bonds or loans, exhibits strongly asymmetric behavior. In this paper, we have analyzed the effective portfolio risk management through a computational state space model by using particle filter through sequential estimation of volatility. The computational model comprises with Extended weight Moving Average Model and Black Scholes-Option Pricing model as well as GARCH deterministic volatility model. The outcome of the model establishes the effectiveness of particle filter for estimating volatility of call option prices for future portfolio returns and it can able to predict the investor's financial risk and measures in a significant manner.

Keywords: Portfolio, financial risk, volatility, particle filter, call option, put option.

1. Introduction

The volatility of a stock is defined as the measure of variation of price of a financial instrument over a time period . When the time period of interest is one year, then the volatility is an annual volatility α_{year} and when the time period of interest is one day, then the volatility is a daily volatility Whatsoever, annual volatility is frequently estimated by first estimation daily volatility using daily log stock returns data. The three main purposes of Estimating volatility are for risk management, for asset allocation, and for taking bets on future volatility. A large part of risk management is measuring the potential future losses of a portfolio of assets, and in order to measure these potential losses, estimates must be made of future volatilities and correlations

The Black-Scholes partial deferential equation and ultimately solve the equation for a European call option In the BSOPM (Black Scholes-Option Pricing model) framework, the annual volatility is taken as constant. It employs a common method which simply calculates the sample standard deviation of the daily log returns of the stock over the past N days by using the equations as below:

$$S_t = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(R_i - \overline{R})^2} \tag{1}$$

Where the average value of the stock return is given as

$$\overline{R} = \frac{1}{n}\sum_{i=1}^{n} R_i$$

Where n is the number of stock return and

$$R_i = \ln(S_i / S_{i-1}) \tag{2}$$

S_i gives us an estimate of daily volatility. σ_t . Since α_{year} is an annual quantity; we have to scale or estimate St. α_{year} which is estimated by

$$\sigma_{\text{year}} \approx \frac{s_t}{\sqrt{1/\text{TD}}} \tag{3}$$

Where TD is the annual number of Trading Days (TD)

To simulate return values for testing the methods, we will use the stochastic differential equation that corresponds to geometric Brownian motion,

$$\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dX$$

where $S(t)$ is the stock price at time t, μ is a measure of the average rate of growth of the asset price, dt is the change in time, σ is the volatility, and dX is known as a Wiener process because it is a random normal variable with a mean of zero and a standard deviation of dt

For the numerical simulation the initial asset price was set equal to p, Where p is a numerical value. In terms of the model $S_0 = p$, and St is the closing price f

We will assume that we can obtain a closing stock price for 365 consecutive days

$$S_{t+1} = S_t + St_i \cdot \mu \cdot dt + S_t \cdot \sigma \cdot dX$$

The sample standard deviation St, provides a very simple tool for estimating daily volatility σ_t, since it assigns equal weight to each daily log return R_t. Apart from this, we can also utilize quite more accurate weighted techniques Like ARCH and GARCH models. From equation (1) R_{t-i} which is defined as the continuously compounded return on the stock during day t-i.. Squaring σ_t and St of the conditional daily variance on day t, using the most recent N observations of u, we can obtain the equation (4) as below:

$$\hat{\sigma}_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}\alpha_i(R_{t-i}-\bar{R})^2$$

$$\hat{\sigma}_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}(R_{t-i}-\bar{R})^2 \qquad (4)$$

We can employ another alluring technique for estimating the conditional daily variance σ_t^2 that involves assigning weights α_i to the most recent observations of u as shown in the equation (5).

$$\hat{\sigma}_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}\alpha_i(R_{t-i}-\bar{R})^2 \qquad (5)$$

Where,

$0 \le \alpha_i \le \alpha_j$, When i>=j

$$\sum_{i=1}^{N}\alpha_i = 1$$

As we agree on a point that the most recent observations must be assign more weight as compare to the earlier observations. The recent observations likely contains more information about the current level of the conditional daily variance σ_t^2 . The total Weight of all observation must be hundred percentage i.e one. Further this idea can be extended by adding a long run average VL in the equation (5). The long run average must have a weight γ as specified in equation (6).

$$\hat{\sigma}_t^2 = \gamma V_L + \frac{1}{n-1}\sum_{i=1}^{n}\alpha_i(R_{t-i}-\bar{R})^2 \qquad (6)$$

When

$\gamma + \sum_{i=1}^{N}\alpha_i = 1$ and $0 \le \alpha_i \le \alpha_j$, When i>=j

If we replace $\bar{u}$ in equation (6) by the true long-term "true" mean μ_u of the return U_t, then we can able to obtain a deterministic expression for the true conditional daily variance σ_t^2 . This leads to Engle's ARCH(N) model, where the weights again sum to unity and the expressional representation is as below:

$$\hat{\sigma}_t^2 = \gamma V_L + \frac{1}{n-1}\sum_{i=1}^{n}\alpha_i(R_{t-i}-\mu_u)^2 \qquad (7)$$

Where $\gamma + \sum_{i=1}^{N}\alpha_i = 1$

μ_u can be considered to be zero. The Bollerslev's GARCH approach which extends the idea of Engle's ARCH approach in equation (7) by applying true conditional daily variance from past days into the deterministic expression for the true conditional daily variance of the current days. The GARCH (P,Q) model may be defined as in the equation (8, 9, 10)

$$\epsilon_t = \sigma_t z_t; \qquad (8)$$

$$u_t = \mu_u + \epsilon_t \qquad (9)$$

Where $\hat{\sigma}_t^2 = \gamma V_L + \sum_{i=1}^{P}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{Q}\beta_j\sigma_{t-j}^2 \qquad (10)$

$$\gamma + \sum_{i=1}^{N}\alpha_i + \sum_{j=1}^{M}\beta_j = 1;$$

$$\gamma V_L > 0;$$

$$\alpha_i \ge 0; \text{for } i = 1,2,...,N;$$

$$\beta_j \ge 0, \text{ for } j = 1,2,...,M.$$

The term ϵ_t is a zero mean random disturbance, or stock , in the mean μ_u of Ut. The equation (9) is known as the conditional daily mean model and equation (10) is known as the conditional daily variance model. σ_t^2 can also be viewed as the conditional daily variance of the GARCH disturbance term ϵ_t with the expressional value as:

$$\sigma_t^2 = V_{t|t-1}(u_t) = V_{t|t-1}(\mu_u + \epsilon_t) = E_{t|t-1}(\epsilon_t^2) = V_{t|t-1}(\epsilon_t),$$

The most popular specification of the conditional daily variance for the simple GARCH(P,Q) model is a GARCH(1,1) which is represented as:

$$\sigma_t^2 = \gamma V_L + \alpha \in_{t-1}^2 + \beta\alpha_{t-1}^2, \quad (11)$$

Where $\gamma + \alpha + \beta = 1;$

$\gamma V_L > 0;$

$\alpha, \beta \geq 0$

The exponentially weighted moving average (EWMA) model is a particular case of GARCH(1,1,) model when we set γ=0, $\alpha = 1 - \lambda$ and $\beta = \lambda$ in the equation (11) The EWMA Model with the expression as follows:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda) \in_{t-1}^2 \quad (12)$$

For modeling time varying volatility purposes, utilization of EWMA models can produce more significance.

For estimating volatility from a well recognized model like GARCH(M,N) can be embedded with Black Scholes Option Pricing Model using Auxiliary Particle filter techniques.

The Pricing model for European Style call option with non-divident underlying stock price of Black Scholes Option Price model has the solution for C_t(Call Option at Time t)

$$C_t = S_t N(d1) - Ke^{-r(T-t)} N(d2) \quad (13)$$

Where

$$d1 = \frac{\ln(S_t / Ke^{-r(T-t)}}{\sigma\sqrt{T-t}} + \frac{1}{2}\sigma\sqrt{T-t}$$

$$d2 = \frac{\ln(S_t / Ke^{-r(T-t)}}{\sigma\sqrt{T-t}} - \frac{1}{2}\sigma\sqrt{T-t}$$

here N denotes the cdf of the standard Gussian N(0,1). St is the underlying stock price at time t, r is the risk free annual rate of interest, K is the Strike Price of the call option to be mature T-t.

2. MODEL FRAME WORK

Generally σ_t^2 represents the daily conditional variance of the underlying stock price at day t, whereas St is denoted as, the underlying stock price at day t. Let we define c_t^{obs} as the observed market price of the call option at day t. T is denoted as the maturity date of the call option and r as annual risk free rate of interest.

Now we introduce the following sequential estimation of volatility model framework with system equation and observation equation as follows:

System Equation :

$$\sigma_t^2 = [\lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2] + \eta_t$$

Observation Equation:

$$C_t^{Obs} = [BSOPM(P((S_T \mid S_t : \phi = \sigma), S_t, \sigma_t^2) + \xi_t$$

Where λ=0.94

The errors in the system and observation equations are additive which can be and as such the equations may naturally be expressed as follows.

System Equation :

$$X_t = \ln(\sigma_t^2) = \ln([\lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2] + \eta_t)$$

Observation Equation :

$$Z_t = \ln(C_t^{Obs}) = \ln([BSOPM(P((S_T \mid S_t : \phi = \sigma), S_t, \sigma_t^2) + \xi_t)$$

As per the definition, the system error term η_t which is additive to ensure the non-negativity of the conditional daily variance σ_t^2, required to be non-negative. The observation error term ξ_t is additive so that the observed option price c_t^{obs} is ensured as non-negative and ξ_t helps to enforce Merton's first lower bound, β_1. The additive error terms signifies that the "error" generally scales with the signal which means, on sheer basis, higher values of both σ_t^2 and c_t^{obs} are more prone to higher noise as compared to lower values of σ_t^2 and c_t^{obs}. System noise in equation (10) is represented as η_t and the additive observation noise in equation (11) is represented as ξ_t. In the system equation (10), benchmark Riskmetrics EWMA model is utilized. As mentioned by Siergiy Ladokhin in his thesis with analysis , the EWMA model is a simple and general model and well accepted to a wide range of stock return data.

While developing this Riskmetrics model, analysts at J.P. Morgan has processed 480 financial time series and associated each series with an "optimal" decay factor λ which minimized the root mean squared error (RMSE) of the conditional variance forecast as specified in

[4]. The model employs, RMSE as the forecast error measure criterion. For the daily log stock return data, it was observed that with a decay factor of $\lambda = 0.94$, it can able to yield the optimal results for the given set of time series. Apart from that, It was also discovered that this particular specification of the EWMA model consistently can able to capture various characteristics of daily log stock return data, along with volatility clustering which another advantage of this model. Riskmetrics EWMA model assumes that $z_t \sim N(0,1)$, where $\in_t = \sigma_t z_t$ as per the GARCH (P,Q) model.

The random system error η_t basically captures the nonsystematic biases of the EWMA model. It is intended to account for those errors of the deterministic EWMA model which either varies randomly or non-systematically over a period of time. In the other side, the deterministic volatility model is less capable to capture many of the complex features of stock return data, such as the leverage effect, etc., hence this random error term is required to be included for smooth functioning in the EWMA model. But in practical, heteroskedastic error is more suitably included as opposed to a simple white noise assumption for η_t. In addition, it may be worthwhile to model any systematic elements of the EWMA model error, though it throws more challenging task. Pragmatically modelers may be more eager to use more sophisticated GARCH models, like the E-GARCH model, rather than attempting to model the EWMA model error η_t due to its complex characteristics.

In the observation equation (11), BSOPM is used as a base model for the observed option price, while allowing for a random observation error ξ_t which is accounted for the non-systematic shortcomings of the BSOPM. It is apparent that equation (3) expresses ln (c_t) as a non-linear function of the state $\ln(\sigma_t^2)$. For simplification, the assumption is taken that the random error that processes ξ_t and η_t are basically represented as Gaussian white noise. On the other hand with the leptokurtotic nature of financial data, it seems that a more fat-tailed distribution such as the Student t might be appropriate to yield the appropriate result. That's why, it is apparent that a particle filter is far more appropriate as compared to a Extended Kalman filter, despite this being most basic model, because the particle filter has been designed in a diversified manner which can very effectively able to handle non-linear, non-Gaussian state space models.

3.. ASSUMPTIONS UNDERTAKEN IN THE PROPOSED MODEL

As this model is being passed through a Auxiliary particle filter, definitely, all of the underlying assumptions of the particle filter well utilized here also. Six assumptions are being taken for smooth functioning of the designed model which are specified as follows:

A1 **:** The System error terms $\eta_t \sim$ NIID ($0, \sigma_\eta$) denotes Gussian white noise where σ_η denotes the standard deviation of the system error process.

A2 **:** In the system equation, the Risk metrics Extended Weighted Moving Average generating process Zt ~ N(0,1), where $\in_t = \sigma_t z_t$. That signifies that, the Riskmetrics model is a specification of Gussian GARCH(1,1) model.

A3 **:** The initial pdfs of the two state vector variables are Gaussian ln($\sigma_2{}^2$) ~ N($0, SD_{\ln(\sigma 2)}$) and ln(c_1) ~ N(0, $SD_{\ln(c1)}$ Where $SD_{\ln(\sigma 2)}$ and SD $_{\ln(c1)}$ denote the standard deviation of the initial state vector variables. Here, it has been assumed that the trading takes place in an arbitrage-free environment, where the market is completely efficient with the view that "unusual" trades do not take place. For which, an option equilibrium or true theoretical value Ct is assumed to be more equivalent to its observed market price Ct abs. The BSOPM model error is fully observable, even though it reflects the models divergence from a theoretical option price Ct.

A4**:** Here, it is assumed that there exists a zero correlation between the underlying stock prices St and the BSOPM model error (or observation error). The critical point is that when the underlying stock price data and option price data are not recorded synchronously or the actual trading of the underlying stock necessitate the trading of the associated equity option at that time, this assumption is not valid..

A5**:** The conditional distribution of S_T is lognormal and allows to be expressed in close format

A6**:** In the last assumption, the underlying stock price S_t and the risk-free model are observed without error. Although it can be assumed that errors in the observed

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ISSN (Online): 1694-0814
www.IJCSI.org

values of S_t are already observed in the observation error ξ_t.

4. MODEL SIMULATION

For model simulation, we have to first trace out the specific parameter values and initial values that have been used in this particular simulation process so that simulation becomes easier.

As, it has been observed that, the standard deviation SD_V, of the observation error v_t is defined at a very small quantity i.e. 0.05 which emphasizes our confidence that the error-adjusted BSOPM price c_t probably does not deviate from the observed market price c_t^{obs} by more than roughly 10% i.e. $e^{\pm 3SDV} = e^{\pm 0.10} \approx (0.74,1.12)$. Therefore, an initial value ln (c_t^{obs}) of 1.9 has taken because it can able to finds out the approximate difference of 5% between $c_1^{(obs)}$ and c_1 (where ln (c_1)=1.8) effectively. All the intial values ana parameters are used in the model is listed in the Table 1.

TABLE 1: INITIAL VALUES AND PARAMETERS FOR MODEL SIMULATION

Variable	Value	Meaning
T	256	Maturity of the call option (days).
TD	256	Annual number of trading days.
K	50	Strike price of the call option (Rs).
np	1000	Number of particles per time step (i.e. day).
r	0.02	Risk free rate of annual interest
xl_1	0.0005	Initial value of the first component of the state at day t=1.
$x2_1$	1.8	Initial value of the second component of the state at day t=1.
S_1	55	Initial value of the underlying stock at day t = 1.
μ_u	0.00025	Long-term average of the daily log returns on the underlying stock.
u_1	0.02	Initial value of the daily log return on the underlying stock (between end of day 1 and end of day 0).
z_1	1.9	Initial value of the observation at day *t = 1*.
SD_η	0.05	Standard deviation of first component of system noise.
SD_ξ	0.04	Standard deviation of second component of system noise.
SD_v	0.05	Standard deviation of observation noise.

5. ANALYSIS OF THE SIMULATION RESULT

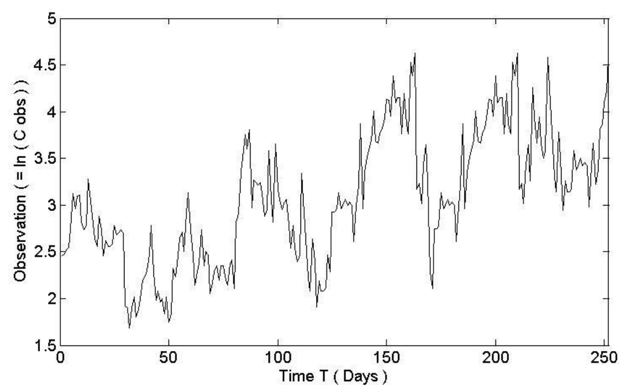

Fig. 1: Daily Log returns and Observed option price

The simulated stock price experiences a general growth trend over the 256 day period although there are sporadic period of decline in S_t as well and they are projected in fig. 1 to fig. 6 with different attributes.

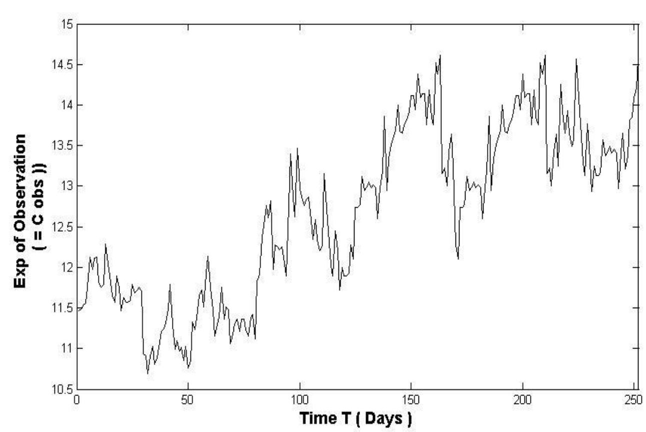

Fig. 2: Daily Log returns and Expected Observed value

The Standard deviation of the observed error is found to be quite small at .05. This confirms our belief that the error adjusted by BSOPM price Ct probably does not deviate from the observed market price Ct^{Obs} by more roughly 10%.

It has been observed that Auxiliary particle filter is effective in tracking the dynamic of both components of the state vector. As our simulated stock does not experience many sharp jumps in its volatility; which proves that the time series is fairly steady in comparison to others.

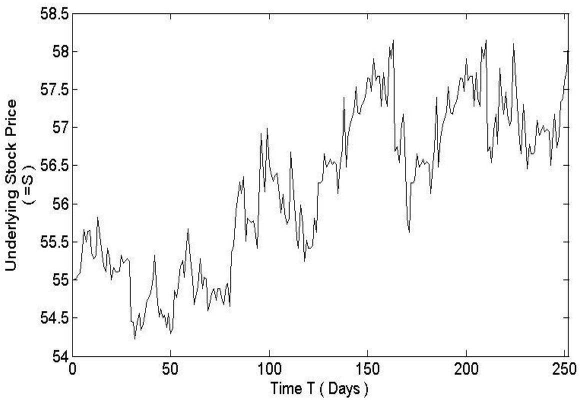

Fig. 3: Daily Log return of Underlying Stock Price

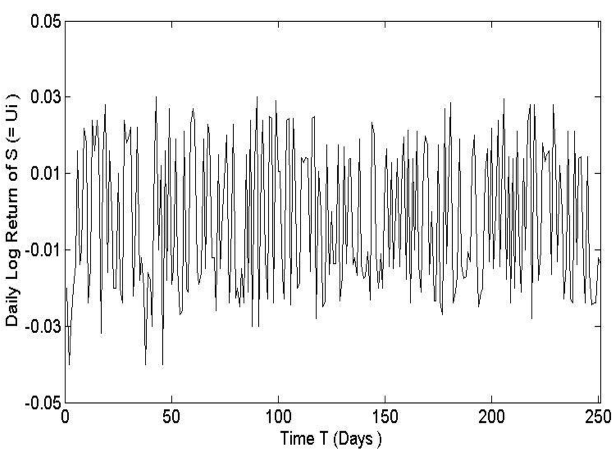

Fig. 4: Daily Log return of System Noise

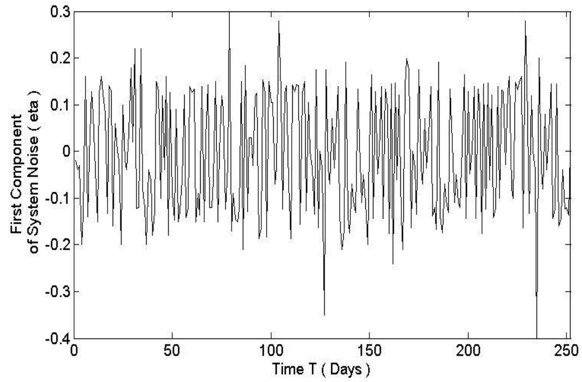

Fig. 5: System Noise of Simulated Model

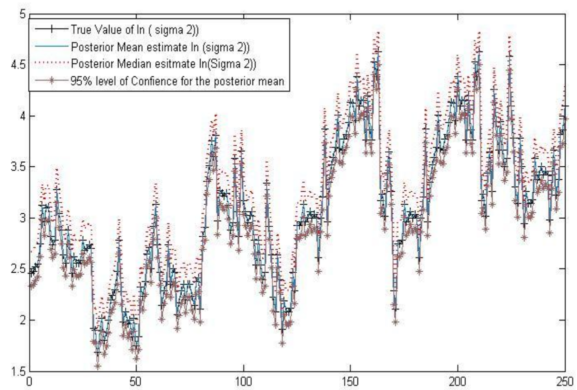

Fig. 6: Observation of Simulated Model

Another fact it has been observed that, the predictive distribution get progressively wider as we move from one step prediction to seven step prediction which is due to the uncertainty of our estimates increases with the forecast horizon. In over 95% of the time steps, the 95% PIs for the one-step, two step, and seven steps predictions can able to confine the true observed value.

6. CONCLUSION

The proposed hybrid model which is a complete equilibrium model of the option pricing problem, provides the final formula, which is the function of observable variables which can effectively utilized for solving problems through particle filter. Rather than assuming that the logarithm of the stock price follows a normal distribution, we assume that the square root of the stock price follows a normal distribution. Due to its mathematical as well as computational characteristics our proposed model in this paper carries a pragmatic alternative for risk analysis of portfolios.

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