

Stability and stabilization of a flexible manipulator

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Abstract

In this paper, the stability and stabilization of a neutral system with time-delay are treated. First, the flexible manipulator is modeled as a neutral time-delay system form. Then, based on the Lyapunov-Krasovskii functional theory, a delay dependant condition is proposed to test studied system stability, which is applied to a flexible manipulator model. The test shows that this flexible manipulator is instable and it should be stabilized. For this reason, other theorem is elaborated to determinate a feed-back control based on linear matrix inequalities (LMI). Finally, simulation results are presented to prove theoretical development.

Keywords: Flexible Manipulator, Stability, Stabilization, LMI, Lyapunov-Krasovskii.

1. Introduction

Flexible manipulator is among the important tools which are rapidly developed in last decades. It is used generally in grinding, polishing and some other manufacturing tasks. However presence of perturbation and disturbances in considered system environment makes system modeling more complicate. Thus, modeling should be linearized around a chosen steady state. In general, there are many possible models for flexible manipulator. Some manipulators are modeled by neutral time-delay systems [1].

The presence of time delay gives a model more similar to real system than non delayed model. However, the

stability and stabilization steps became more complicate and obligate to guarantee security in robot functioning.

For this reason many research are interested to develop delay-dependent [2-5] and delay-independent [6-10] stability and stabilization conditions using Lyapunov-Krasovskii functional approach.

Lyapunov-Krasovskii functional theory has first started for system without neither uncertainties nor control [11] [12], some robust stability conditions based on LMI approach are given. Then, the guaranteed cost control problem for neutral time delay system with feed-back control is investigated. Some papers are interested on stability and stabilization where a linear-quadratic cost function is considered as a performance measure for the closed-loop systems [13] [14] [15] [16] [17]. In the last years, interesting works have been concerned with uncertain neutral time-delay systems stability and stabilization analysis based on Lyapunov-Krasovskii functional theory [18] [19] [20].

The aim contributions of this paper is to determinate a general solution for flexible manipulator model proposed in literature. Then, based on this general solution, flexible manipulator is modeled in neutral time-delay form. After that, stability and stabilization conditions are proposed in term of linear matrix inequalities.

The paper is organized as follows: in section 2 a brief review on stability analysis of neutral time-delay systems is presented. A flexible manipulator model is developed in

Section 3. Section 4 gives stability and stabilization problem based on linear matrix inequalities.

Section 5 shows and analyzes simulation results.

2. Review of Stability analysis for Neutral delay systems

The stability analysis is an essential step for a system control and a diagnosis strategy. Some conditions are derived in literature to guarantying neutral time-delay systems.

It's already cited that there are two classes of stability conditions for neutral systems with time-delay. The first is a delay independent condition. Stability criteria for neutral systems with multiple time-delays are presented in [12]. Using the Lyapunov second method, Park and al. establish a new delay-independent criterion for the asymptotic stability. In these criteria, the derived sufficient conditions are expressed in terms of LMI so that the criteria are less conservative.

In [21], sufficient conditions for the existence of these observers are derived. Using the linear matrix inequality and the linear matrix equality (LME) formulation, independent of delays stability criteria are derived in [21] for proposed observers.

Wang and al. [22] consider the H^∞ dynamic for linear neutral time-delay systems output feedback controller design problem. The approach here is based on Lyapunov functional due to Krasovsii. A sufficient condition is deduced in terms of linear matrix inequalities.

The second conditions are dependent on a delay size.

In fact, in [13], Sun and al. introduce a new form of the Lyapunov functional that contains a triple integral term $\int_{-\tau}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) R \dot{x}(s) ds d\lambda d\theta$. Two integral inequalities are used to derive a new delay-dependent stability criterion without introducing any free-weighting matrices. Using this criterion, a method of designing a stabilizing state feedback controller is also presented.

The paper of Xin and al. [23] deals with the delay-dependent stability criterion and the state observers design problem as well as observer-based stabilization problem for linear neutral delay systems. A delay-dependent stability criterion is developed, which is presented in terms of a feasibility positive definite solution to a linear matrix inequality.

3. Flexible manipulator modeling

3.1 Determination of general solution

From [24], a differential equation of a flexible manipulator free vibration is written as:

$$-\frac{\delta^2}{\delta x^2} \left[EI \frac{\delta^2 y(x,t)}{\delta x^2} \right] = m \frac{\delta^2 y(x,t)}{\delta t^2} \quad 0 \leq x \leq L \quad (1)$$

where $y(x,t)$ is a transition emplacement for each point x and time t , EI is the flexibility and m is the mass.

The limit conditions are as follow:

$$\begin{aligned} 1) y(0,t) &= u_1(t) & 2) \frac{\delta y(x,t)}{\delta x} \Big|_{x=0} &= u_2(t) \\ 3) y(L,t) &= u_3(t) & 4) \frac{\delta y(x,t)}{\delta x} \Big|_{x=L} &= u_4(t) \end{aligned}$$

Lemma 1: general solution of (1) is expressed as:

$$y(x,t) = \phi(t + \alpha x) + \psi(t - \alpha x) \quad (2)$$

$$\text{where } \alpha = \sqrt[4]{-\frac{m}{EI}}$$

Proof: Equation (1) gives:

$$-\frac{\delta^2}{\delta x^2} \left[EI \frac{\delta^2 y(x,t)}{\delta x^2} \right] - m \frac{\delta^2 y(x,t)}{\delta t^2} = 0 \quad 0 \leq x \leq L \quad (3)$$

Consider Laplace transformation, we obtain:

$$\frac{\delta^4 y(x,p)}{\delta x^4} + \frac{m}{EI} p^2 y(x,p) = 0 \quad 0 \leq x \leq L \quad (4)$$

Its characteristic equation can be written as:

$$\zeta^4 - \left(-\frac{m}{EI} \right) p^2 = 0 \quad (5)$$

The solution of equation (5) is:

$$\zeta = \pm \sqrt[4]{-\frac{m}{EI}} \sqrt{p} \quad (6)$$

Hence, the solution of (1) is:

$$\hat{y}(x,p) = e^{x \sqrt[4]{-\frac{m}{EI}} \sqrt{p}} \gamma_1(p) + e^{-x \sqrt[4]{-\frac{m}{EI}} \sqrt{p}} \gamma_2(p) \quad (7)$$

Let's propose: $s = \sqrt{p}$

Then, the solution can be rewritten as:

$$\hat{y}(x, s) = e^{x\sqrt{\frac{m}{EI}}s} \gamma_1(s^2) + e^{-x\sqrt{\frac{m}{EI}}s} \gamma_2(s^2) \quad (8)$$

Consider two functions $\phi(s)$ and $\psi(s)$ expressed by:

$$\phi(s) = \gamma_1(s^2) \text{ and } \psi(s) = \gamma_2(s^2)$$

Thus, solution can be expressed:

$$\hat{y}(x, s) = e^{x\sqrt{\frac{m}{EI}}s} \phi(s) + e^{-x\sqrt{\frac{m}{EI}}s} \psi(s) \quad (9)$$

Applying Laplace inverse, the general solution (2) is obtained.

3.2 Flexible manipulator model

In this section, modeling is based on first equation and we propose that $Y(t) = y(L, t)$.

Condition 3) gives:

$$Y(t) = \phi(t + \alpha L) + \psi(t - \alpha L) = u_3(t) \quad (10)$$

From condition 1), we have:

$$\phi(t) + \psi(t) = u_1(t) \quad (11)$$

Hence,

$$Y(t - \alpha L) - \psi(t - 2\alpha L) + \psi(t) = u_1(t) \quad (12)$$

In other hand, condition 2) gives:

$$b(\dot{\phi}(t) - \dot{\psi}(t)) = u_2(t) \quad (13)$$

Then,

$$\dot{Y}(t - \alpha L) - \dot{\psi}(t - 2\alpha L) - \dot{\psi}(t) = \frac{1}{\alpha} u_2(t) \quad (14)$$

Condition 4) gives:

$$\alpha(\dot{\phi}(t + \alpha L) - \dot{\psi}(t - \alpha L)) = u_4(t) \quad (15)$$

Hence,

$$\dot{Y}(t) - 2\dot{\psi}(t - \alpha L) = \frac{1}{\alpha} u_4(t) \quad (16)$$

Then,

$$\dot{Y}(t - \alpha L) = 2\dot{\psi}(t - 2\alpha L) + \frac{1}{\alpha} u_4(t - \alpha L) \quad (17)$$

Substituting $\dot{Y}(t - \alpha L)$ into (14), we obtain:

$$2\dot{\psi}(t - 2\alpha L) + \frac{1}{\alpha} u_4(t - \alpha L) - \dot{\psi}(t - 2\alpha L) - \dot{\psi}(t) = \frac{1}{\alpha} u_2(t) \quad (18)$$

Therefore,

$$\dot{\psi}(t - 2\alpha L) - \dot{\psi}(t) = \frac{1}{\alpha} (u_2(t) - u_4(t - \alpha L)) \quad (19)$$

with addition of (19) and (12) and when we replace $Y(t - \alpha L)$ by $u_3(t - \alpha L)$, we obtain a neutral time-delay system.

$$\dot{\psi}(t) = \psi(t) - \psi(t - 2\alpha L) + \dot{\psi}(t - 2\alpha L) - \frac{1}{\alpha} (u_2(t) - u_4(t - \alpha L)) + u_3(t - \alpha L) - u_1(t) \quad (20)$$

4. Stability and stabilization analysis

4.1 Stability conditions

In this paper, a condition of stability for a type of neutral system with delayed control will be presented.

Consider a neutral time-delay system with the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t - h) + A_d \dot{x}(t - d) + Bu(t) + B_1 u(t - \tau_1) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \quad (21)$$

The following theorem shows a new condition for system (21) with robust delayed control.

Theorem 4.1

Consider neutral system (21) with given constants $h^* > 0$. h, d and τ_1 are supposed different. System (21) is asymptotically stable for any $0 < \tau \leq h^*$, if there exist matrices $X > 0$, $T > 0$, and $Y > 0$ satisfy the following LMI:

$$\begin{pmatrix} G(X, A, A_h) & 0 & A_d Y & B & B_1 & XA^T & XC^T & X & A_h Y \\ * & -T & 0 & 0 & 0 & TA_h^T & 0 & 0 & 0 \\ * & * & -Y & 0 & 0 & YA_d^T & 0 & 0 & 0 \\ * & * & * & I & 0 & B^T & 0 & 0 & 0 \\ * & * & * & * & I & B_1^T & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{1+h^*}Y & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -T & 0 \\ * & * & * & * & * & * & * & * & -\frac{1}{h^*}Y \end{pmatrix} < 0 \quad (22)$$

where: “*” and I denote respectively the transposed elements in the symmetric position and identity matrix.

$$G(X, A, A_h) = (A + A_h)X + X(A + A_h)^T$$

Proof:

Consider the vector $U(t)$ and the matrix \tilde{B} expressed as:

$$U(t) = \begin{pmatrix} u(t) \\ u(t - \tau_1) \end{pmatrix} \text{ and } \tilde{B} = (B \quad B_1).$$

The system (21) becomes:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h) + A_d \dot{x}(t-d) + \tilde{B}U(t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \quad (23)$$

Hence, the result follows immediately by applying Theorem 2.2 in [25].

4.2 Stabilization

In this section neutral system will be stabilized using a feed back control to guarantying system stability. The time delay in control can caused system instability.

Considered system with a feed-back control $w(t) = Kx(t)$ can be written as:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h) + A_d \dot{x}(t-d) + Bu(t) + B_1 u(t - \tau_1) + B_w w(t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \quad (24)$$

To achieve stabilization, feed-back control should be determined.

Hence, system (24) can be written as:

$$\begin{cases} \dot{x}(t) = (A + B_w K)x(t) + A_h x(t-h) + A_d \dot{x}(t-d) + Bu(t) + B_1 u(t - \tau_1) \\ y(t) = Cx(t) \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \quad (25)$$

This system has the same form of system (21). $w(t)$ guarantees system stabilization only if system (25) is stable. The following theorem gives stability condition if system (25).

Theorem 4.2:

Consider neutral system (25) with given constants $h^* > 0$. h , d and τ_1 are supposed different. System (25) is asymptotically stable for any $0 < \tau \leq h^*$, if there exist matrices $X > 0$, $T > 0$, and $Y > 0$ and matrix F satisfy the following LMI:

$$\begin{pmatrix} G(X, A, A_h) & 0 & A_d Y & B & B_1 & (AX + B_w F)^T & XC^T & X & A_h Y \\ * & -T & 0 & 0 & 0 & TA_h^T & 0 & 0 & 0 \\ * & * & -Y & 0 & 0 & YA_d^T & 0 & 0 & 0 \\ * & * & * & I & 0 & B^T & 0 & 0 & 0 \\ * & * & * & * & I & B_1^T & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{1+h^*}Y & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -T & 0 \\ * & * & * & * & * & * & * & * & -\frac{1}{h^*}Y \end{pmatrix} < 0 \quad (26)$$

where: “*” and I denote respectively the transposed elements in the symmetric position and identity matrix.

$$(A + A_h)X + X(A + A_h)^T + B_w F + F^T B_w^T$$

where $w(t) = FX^{-1}x(t)$.

Proof:

The result follows immediately by applying Theorem III.1 to the closed-loop system (25) and setting $F = KX$.

5. Stability and stabilization conditions application for a flexible manipulator

Consider system (20) with $2\alpha L = 1$, the fault and output matrices are respectively $F = [1 \quad 1]$ and $C = [1 \quad 1]$

and the fault vector to be detected is $f = [f_1 \quad f_2]^T$ where

$$f_1(t) = \begin{cases} 1, & 10 \leq t \leq 25 \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{and } f_2(t) = \begin{cases} 2, & 30 \leq t \leq 45 \\ 0, & \text{otherwise.} \end{cases}$$

In this study, we suppose that the faults do not occur simultaneously and their modes are unknown.

First, we start with stability analysis test. The application of theorem III.1 and by solving the LMI (22), the solution is infeasible. Hence, system (21) is non stable which justify the free vibration of manipulator. This result is verified by the state response behavior of nominal system Fig.1.

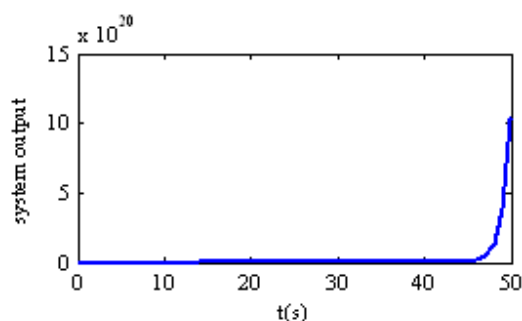


Fig.1 The state response behavior of nominal system

In order to guarantee the good functioning of robot, the manipulator stabilization should be achieved.

Consider $B_w = 1$.

By Theorem III.2, the feasibility of LMI (26) is obtained with:

$$X = 1.1790, Y = 2.1933, T = 0.3456$$

et $F = -4.1962$.

With the control law $w(t) = -3.559x(t)$ Fig.3, the closed-loop system is asymptotically stable and the response of the closed-loop system is illustrated by Fig.2.

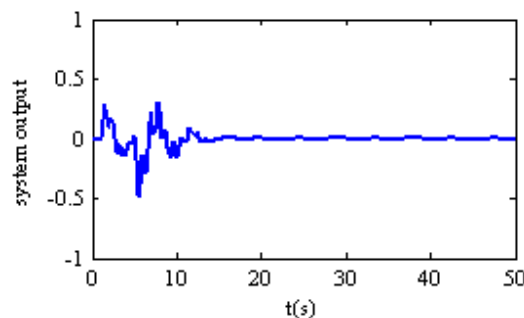


Fig.2 The state response of augmented system

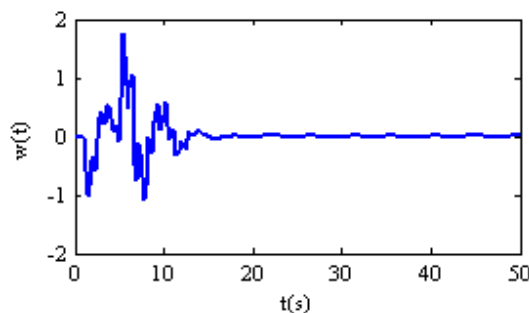


Fig.3. feed-back control evolution

System output behavior compose two part; first part is between initial time 0s to 15s and it is characterized by a little vibration or perturbation caused by feed-back control behavior. Second time is from 15s, the system is stable. We can conclude that control can be a motor which make some vibration in first time and after that the manipulator is fixed.

6. Conclusion

Flexible manipulator is modeled in neutral time-delay form using a general solution which is determinate first. Then, stability and stabilization conditions are obtained in term of linear matrix inequalities. Developed theories are applied to a flexible manipulator. Hence, this manipulator is instable due to free vibrations. Stabilization condition gives a feed back control which stabilizes the functioning of manipulator. This control can represent the same compartment of a motor. When the flexible manipulator model is stabilized, the fault detection and isolation of this system can be achieved to reduce accidents caused by faulty flexible manipulator.

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