On the sensitivity of a discrete linear system with perturbed dynamics

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Abstract
In this work, we consider a linear discrete system where a part of dynamics is affected by a disturbance. Being unable to cancel the effects of this disturbance, we propose a control law in closed loop to reduce the sensitivity of the system output to the disturbance. Finally, and to illustrate the results, we give examples in which it is based on the technique of pole placement. All simulations are done using Matlab/Simulink TM
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1. Introduction

When modeling a system, the mutual connections that bind with its environment can't be ignored and so, we are often obliged to take into account certain undesirable parameters, let's mention as examples pollution, bacterial infection, earthquakes, severe weather…In order to face such problems, scientists have elaborated different approaches such as the identifiability (Arild Thomson 2007; Kauffman et al., 2004; Robert et al., 2007)and the detectability (Franklin, 2001; Kailath , 1980; Ogata, 1995) or the H_∞ control theory (Chi Tsong, 2008; Dingyu Xie, 2002; Goodwin, 2007), the theory of sentinel (Lions et al., 1986; Lions, 1988) and the frequency domain and robustness (Ackermann, 1993; Ayala et al., 2002; Rosario, 2007; Gu et al., 2005). Our contribution in this direction is to construct, under certain hypothesis, a control law so that the sensitivity of the resulting system output would be relatively tolerable.

More precisely, let's consider the following discrete linear perturbed system

$$\begin{cases} x_{i+1} = (A+\Delta A)x_i + Bu_i \\ x_0 \end{cases} \qquad (1)$$

The corresponding output is $y_i = Cx_i$ (2)

where A, B and C are respectively (nxn), (nxm) and (pxn) matrices and the disturbance ΔA has the form $\Delta A=\alpha E$ with $\alpha \in [\alpha_{\min} \ \ \alpha_{\max}]$ is unknown and E a (nxn) matrice which defines the emplacement of the disturbance in the system dynamics.

To attenuate the effect of the perturbation, we investigate a control law

$$u_i = Ky_i$$

in such a way that the resulting output of the system verifies

$$\left\| \frac{\partial y_i}{\partial \alpha} \right\| < \varepsilon, \ \forall i \geq 0 \, and \, \alpha \in [\alpha_{\min} \ \ \alpha_{\max}] \qquad (3)$$

where ε is a predefined tolerance threshold and where the inequality

$$\left\| \frac{\partial y_i}{\partial \alpha} \right\| \prec \varepsilon$$

means that the sensitivity of y_i to the perturbed α will not reach the predefined threshold ε.

2. Some technical results

To understand this problem, we limit ourselves to the uncertainty in one of the parameters of the matrix A, so

$$E = \begin{pmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 0 \end{pmatrix} \qquad (4)$$

substituting the control $u_i=Ky_i$ in the equation (1) hence the new discrete linear system of the closed loop can be rewritten as follows

$$\begin{cases} x_{i+1} = (A+\alpha E+BKC)x_i \\ y_i = Cx_i \end{cases} \quad (5)$$

let's compute the derivative of x_{i+1} with respect to α, we will have

$$\frac{dx_{i+1}}{d\alpha} = A\frac{dx_i}{d\alpha} + Ex_i + \alpha E\frac{dx_i}{d\alpha} + BKC\frac{dx_i}{d\alpha} \quad (6)$$

we put

$$z_i = \frac{dx_i}{d\alpha} \quad (7)$$

so, we will have

$$z_{i+1} = \frac{dx_{i+1}}{d\alpha} = (A+\alpha E+BKC)z_i + Ex_i \quad (8)$$

let's put

$$\phi = A+\alpha E+BKC$$

then, we can write

$$z_i = \phi^i z_0 + \sum_{j=1}^{i}\phi^{i-j}Ex_{j-1}$$

since $z_0 = 0$, it's easy to show that

$$z_i = \sum_{j=1}^{i}\phi^{i-j}E\phi^{j-1}x_0$$

and then the previous inequality can be written as

$$\frac{dy_i}{d\alpha} = C\sum_{j=1}^{i}\phi^{i-j}E\phi^{j-1}x_0 \quad (9)$$

we assume that the gain $K \in \mathbb{R}^{m x n}$ is such that

$$\rho(A+\alpha_{\max}E+BKC) \prec 1 \quad (10)$$

where $\rho(A+\alpha_{\max}E+BKC) \prec 1$ is the spectral radius of the matrix $A+\alpha_{\max}E+BKC$.

then, it is known (see Burden et al., 2001; James et al., 2007) that for any $\varepsilon_0 > 0$, there exists norm $\|.\|_{\max}$ such as

$$\|A+\alpha_{\max}.E+BKC\|_{\max} < \rho+\varepsilon_0 \quad (11)$$

we recall that if

$$H = A+\alpha_{\max}E+BKC$$

according to Jordan transformation $H = SJS^{-1}$

and so, the matrix norm $\|.\|_{\max}$ is

$$\|H\|_{\max} = \|T^{-1}S^{-1}HST\|_{\infty} \quad (12)$$

where

$$T = \begin{pmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & \varepsilon_0^{-1} & 0 & \dots & 0 \\ 0 & 0 & \varepsilon_0^{-2} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \varepsilon_0^{1-n} \end{pmatrix} \quad (13)$$

and where the matrix norm $\|.\|_{\infty}$ is given by

$$\|P\|_{\infty} = \max_{1<i<n}\sum_{j}^{n}|p_{ij}|$$

and the vector norm is

$$\|x\|_{\max} = \|T^{-1}S^{-1}x\|_{\infty}; \forall x \in \mathbb{R}^n \quad (14)$$

according to (9), we can write

$$\left\|\frac{dy_i}{d\alpha}\right\| \le \sum_{j=1}^{i}\|C\|\|\phi\|_{\max}^{i-j}\|E\|_{\max}\|\phi\|_{\max}^{j-1}\|x_0\|_{\max}$$

$$\le \sum_{j=1}^{i}\|C\|\|\phi\|_{\max}^{i-1}\|E\|_{\max}\|x_0\|_{\max}$$

hence,

$$\left\|\frac{dy_i}{d\alpha}\right\| \le \sum_{j=1}^{i}\|C\|\|E\|_{\max}\|x_0\|_{\max} i\phi^{i-1} \quad (15)$$

since $\alpha \in [\alpha_{\min} \quad \alpha_{\max}]$, we have $\alpha = p\alpha_{\min} + q\alpha_{\max}$

where $p+q=1$ *and* $p,q \ge 0$

$$\|\phi\| = \|A+(p\alpha_{\min}+q\alpha_{\max})E+BKC\|$$

let's put

$$\Delta_{\min} = A+\alpha_{\min}E+BKC$$

$$\Delta_{\max} = A+\alpha_{\max}E+BKC$$

so, we have

$$\|\phi\| \le \|p\Delta_{\min}+q\Delta_{\max}\| \le p\|\Delta_{\min}\|+q\|\Delta_{\max}\|$$

$$\|\phi\| \le p\|\Delta_{\min}+\Delta_{\max}-\Delta_{\max}\|+q\|\Delta_{\max}\|$$

$$\|\phi\| \le p\|(\alpha_{\min}-\alpha_{\max})E+\Delta_{\max}\|+q\|\Delta_{\max}\|$$

$$\|\phi\| \le |\alpha_{\min}-\alpha_{\max}|\|E\|_{\max}+\|\Delta_{\max}\|$$

and finally, we obtain

$$\|\phi\|_{\max} \le \|A+\alpha_{\max}E+BKC\|_{\max} + |\alpha_{\min}-\alpha_{\max}|\|E\|_{\max} \quad (16)$$

the expression (15) gives

$$\left\|\frac{dy_i}{d\alpha}\right\| \le \|C\|\|E\|_{\max}\|x_0\|_{\max} i\Gamma^{i-1}, \ \forall i \succ 1 \quad (17)$$

where

$$\Gamma = \|A+\alpha_{\max}E+BKC\|_{\max} + |\alpha_{\min}-\alpha_{\max}|\|E\|_{\max}$$

using (11), we have $\forall i > 1$

$$\left\|\frac{dy_i}{d\alpha}\right\| \le \|C\|\|E\|_{\max}\|x_0\|_{\max} i(\rho+\varepsilon_0+|\alpha_{\min}-\alpha_{\max}|\|E\|_{\max})^{i-1} \quad (18)$$

let's put

$$\Psi = \rho+\varepsilon_0+|\alpha_{\min}-\alpha_{\max}|\|E\|_{\max} \quad (19)$$

in the next, we suppose

$$\rho+\varepsilon_0+|\alpha_{\min}-\alpha_{\max}|\|E\|_{\max} < 1 \quad (20)$$

and if we consider the function F defined by

$$F(x) = x\Psi^{x-1} \quad \forall x \geq 1 \qquad (21)$$

it is easy to establish that the F(x) is a decreasing function with the following maximum

$$\max(F(x)) = \omega = -\frac{1}{\ln(\Psi)\exp(1+\ln(\Psi))} \qquad (22)$$

this is reached when

$$x = -\frac{1}{\ln(\Psi)}$$

that means $F(x) \leq \omega, \ \forall x \geq 1$

therefore, we obtain

$$\left\|\frac{dy_i}{d\alpha}\right\| \leq \|C\|\|E\|_{\max}\|x_0\|_{\max}\,\omega \qquad (23)$$

so, in order to realize the sensitivity condition

$$\left\|\frac{\partial y_i}{\partial \alpha}\right\| \prec \varepsilon, \ \forall i \geq 0 \ and \ \alpha \in [\alpha_{\min} \ \ \alpha_{\max}]$$

it is sufficient to verify the ε-tolerance condition

$$\|C\|\|E\|_{\max}\|x_0\|_{\max}\left(-\frac{1}{\ln(\Psi)\exp(1+\ln(\Psi))}\right) \leq \varepsilon \qquad (24)$$

3. Solution of the problem

Using the technical results established in the previous section, we have

3.1 Proposition

If the gain matrix $K \in \mathbb{R}^{m\,x\,n}$ satisfies the two following hypotheses:

h1) $\rho(A+\alpha_{\max}E+BKC) < 1$;

h2) $\rho+\varepsilon_0+|\alpha_{\min}-\alpha_{\max}|\|E\|_{\max} < 1$

$\varepsilon_0 > 0$ is such that $\rho+\varepsilon_0 < 1$

and $\rho(A+\alpha_{\max}E+BKC) < 1$.

then the output y_i resulting from the control law $u_i = Ky_i$ verifies the sensitivity condition

$$\left\|\frac{\partial y_i}{\partial \alpha}\right\| \prec \varepsilon, \ \forall i \geq 0 \ and \ \alpha \in [\alpha_{\min} \ \ \alpha_{\max}]$$

if the following sufficient condition

$$\|C\|\|E\|_{\max}\|x_0\|_{\max}\left(-\frac{1}{\ln(\Psi)\exp(1+\ln(\Psi))}\right) \leq \varepsilon \ \ u_i = Ky_i$$

is realized.

in order to construct a gain matrix $K \in \mathbb{R}^{m\,x\,n}$ verifying the two hypotheses of the last proposition , assuming the controllability of $(A+\alpha_{\max}E, B)$, then it follows from the Ackermann's theorem the existence of $L \in \mathbb{R}^{m\,x\,n}$ such that the poles of the matrix $(A+\alpha_{\max}E+BL)$ may be placed at any desired locations. Consequently, we use the Ackermann's theorem to determine a gain matrix $L \in \mathbb{R}^{m\,x\,n}$ such that $\rho(A+\alpha_{\max}E+BKC) < 1$.

To determine the gain matrix K, we need the following lemma:

3.2 Lemma

If we consider $L \in \mathbb{R}^{m\,x\,n}$, the two following assertions are equivalent

i) $\exists K \in \mathbb{R}^{m\,x\,p} \ such\,that \ L = KC$

ii) $Ker\,C \subset Ker\,L$

3.3 Remark

If $rank(C) = p, then\,Ker\,C^T = \{0\}$, consequently, the (pxp) matrix (CC^T) is invertible and the gain K can be calculated using

$$K = LC^T(CC^T)^{-1}$$

and so, it is the unique solution of the matrix equation

$$L = KC$$

4. Illustrative examples

4.1 Example 1

Let's consider a perturbed discrete linear system

$$\begin{cases} x_{i+1} = \begin{pmatrix} 0.1 & 0.3 \\ 1 & 0.2 \end{pmatrix} x_i + \alpha \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.2 \end{pmatrix} u_i \\ \alpha = [0.1 \quad 0.5] \end{cases}$$

augmented with the output

$$y_i = \begin{pmatrix} 1 & 1 \end{pmatrix} x_i$$

then the system parameters are

$$A = \begin{pmatrix} 0.1 & 0.3 \\ 1 & 0.2 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0.2 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 \end{pmatrix} \ and \ \ E = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$\alpha_{\min} = 0.1, \ \alpha_{\max} = 0.5, \ n = 2, \ m = 1 \, and \ \ p = 1$

we investigate the feedback control $u_i = Ky_i$ such as

$$\left\|\frac{dy_i}{d\alpha}\right\| < 0.4, \ \forall i \geq 0 \ and \ \forall \alpha \in [0.1 \quad 0.5]$$

Step 1

Using Ackermann's method, there exists a gain L such as

$$\rho(A + \alpha_{\max} E + BL) = 0.2$$

with Matlab/Simulink, we find

$$L = (-6 \quad -3)$$

Step 2

Since (CC^T) is invertible, it follows from the previous remark that

$$K = LC^T (CC^T)^{-1} = -4.5$$

then the control u_i can be written such as $u_i = Ky_i$

let's simulate the system output for several values of perturbed parameter α : (0.1 < α < 0.5)

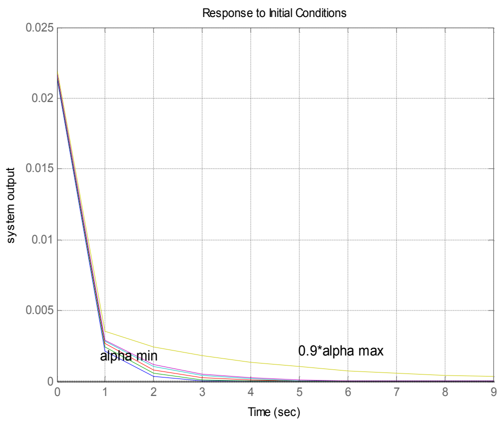

Fig.1. system outputs for multiple disturbance parameter

Figure 1schows that for the control $u_i = Ky_i$, the resulting outputs of the perturbed system are almost confounded, we have taken

$$\alpha_1 = \alpha_{\min} = 0.1,\ \alpha_2 = 0.2,\ \alpha_3 = 0.3$$

$$\alpha_4 = 0.35,\ \alpha_5 = 0.4 \ \textit{and}\ \alpha_6 = 0.45 < \alpha_{\max}$$

this shows therefore, the insensitivity of the system output to disturbance variations. However, by choosing a parameter $\alpha > \alpha_{\max}$, we notice that the resulting output deviates more from the package.

4.2 Example 2

The amount of solute (drug or metabolite) introduced in the human body is often assumed to be stored in different compartments of the body. A separate equation for each compartment relates the rate of solute removable to the amount or concentration of the solute in the compartment.

The solute can be either be transported to another compartment or eliminated from the body by metabolism or excretion. Consider the linear compartment model described in Riggs (1970) for describing the quantities of iodine in humans, where k_{12}, k_{21}, k_{13}, k_{31}, k_{43}, k_u and k_f are the rate constants governing the transfer of iodine between the compartments and its excretion from the body.

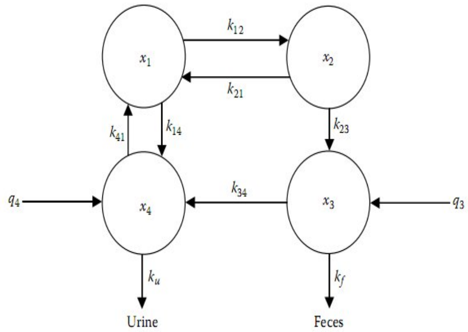

Fig. 2 compartmental model for iodine distribution in a human.

The state variables are: x_1: Amount of inorganic iodine in the thyroid gland, x_2: Amount of organic iodine in the thyroid gland, x_3: Amount of hormonal iodine in the extra thyroidal tissue, x_4: Amount of iodine in the inorganic iodine compartment. And the inputs are: q_3: Rate of entry exogenous iodine, q_4: Rate of entry of exogenous hormonal iodine. The state equations of this system are [see 19]

$$\begin{cases} \dfrac{dx_1}{dt} = -(k_{12} + k_{14})x_1 + k_{21}x_2 + k_{41}x_4 \\ \dfrac{dx_2}{dt} = k_{21}x_1 - (k_{21} + k_{23})x_2 \\ \dfrac{dx_3}{dt} = k_{23}x_2 - (k_{34} + k_f)x_3 + q_3 \\ \dfrac{dx_4}{dt} = k_{14}x_4 + k_{34}x_3 - (k_{14} + k_u)x_4 + q_4 \end{cases}$$

and the corresponding outputs are

$$\begin{cases} y_1 = x_1 + x_2 + x_3 + x_4 \\ y_2 = k_f x_3 + k_u x_4 \end{cases}$$

so, the matrices of the system are

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}, B=\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$C=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & k_f & k_u \end{pmatrix}$$

where

$$a_{11}=-(k_{12}+k_{14}),\ a_{12}=k_{21},$$
$$a_{13}=a_{14}=0,$$
$$a_{21}=k_{12},\ a_{22}=-(k_{21}+k_{23}),$$
$$a_{23}=a_{24}=0,$$
$$a_{31}=a_{34}=0,$$
$$a_{32}=k_{23},\ a_{33}=-(k_{34}+k_f),$$
$$a_{41}=k_{14},\ a_{42}=0,$$
$$a_{43}=k_{34,}\ a_{44}=-(k_{14}+k_u)$$

Baseline values of the system parameters are

$k_{12}=0.8/day,\ k_{21}=0.05/day, k_{23}=0.01/day$

$k_{34}=0.3/day,\ k_{14}=0.15/day, k_{41}=0.5/day$

$k_f=0.02/day,\ k_u=1.2/day$

We suppose that the dynamic of the system is affected by a disturbance only in the parameter a_{11}

so, the perturbed parameter a_{11}^p becomes

$$a_{11}^p=a_{11}+\alpha$$

we are first interested in determining a control law such that the output y_1 is insensitive to the disturbance α in the system dynamic.

otherwise

$$\left\|\frac{dy_1}{d\alpha}\right\|\le\varepsilon_1, \forall i\ge 0 \, and \, \forall\alpha\in[0.001 \quad 0.145]$$

where ε_1 is the predefined threshold tolerance, it has the value

$$\varepsilon_2=0.04$$

then, the same work can be applied to the second output y_2 with a predefined threshold tolerance ε_2

$$\varepsilon_2=0.04$$

so, we have

$$E=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$n=4, m=2 \ \ and \ \ p=2$$

Step 1

First, we will discretize our continuous system using a sampling period $T_e = 1$ sec.

consequently, the new discrete linear system is written as

$$\begin{cases} x_{i+1}=A_d x_i+\alpha E x_i+B_d u_i \\ y_i=C_d x_i \end{cases}$$

the matrices of the new discrete linear system are calculated using Matlab/Simulink

$$A_d=\begin{pmatrix} 0.8788 & 0.0047 & 0.0386 & 0.2209 \\ 0.7422 & 0.9780 & 0.0210 & 0.1154 \\ 0.0034 & 0.0084 & 0.7292 & 0.0004 \\ 0.0665 & 0.0010 & 0.1197 & 0.1951 \end{pmatrix}$$

$$B_d=\begin{pmatrix} 0.01519 & 0.1450 \\ 0.0033 & 0.0439 \\ 0.8558 & 0.0001 \\ 0.0819 & 1.4863 \end{pmatrix}$$

and

$$C_d=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0.02 & 1.2 \end{pmatrix}$$

Step 2

Then, the system will be simulated for several disturbance parameters

$\alpha=[0.0010 \ \ 0.0015 \ \ 0.0075 \ \ 0.0540 \ \ 0.0810 \ 0.160]$

and $\alpha_{\max}=0.145$

the initial vector is

$x_0=10^{-5}[0.1 \ \ 0.4 \ \ 0.78 \ \ 0.65]$

Step 3

Using Ackermann's method and Matlab/Simulink, we find

$$L=\begin{pmatrix} -9.83 & -42.5612 & 1.3215 & -1.33 \\ 12.2514 & 19.208 & 0.1296 & 2.1322 \end{pmatrix}$$

with $\rho(A_d+\alpha_{\max}E+B_d L)=0.6$

Step 4

We can calculate the gain K using the expression given in the previous remark

$$K=\begin{pmatrix} -17.2177 & 13.4929 \\ 10.6354 & -7.23 \end{pmatrix}$$

so, we will introduce a law control $u_i=Ky_i$ such that

$$\left\|\frac{dy_1}{d\alpha}\right\|\le\varepsilon_1 \ \ and \ \ \left\|\frac{dy_2}{d\alpha}\right\|\le\varepsilon_2$$

Figure 3 illustrates the same interpretations as in the first example, As the value of α remains bounded between α_{min} and α_{max}, and as a condition of tolerance (24) is always true it is always possible to reduce the sensitivity of the output by using a control law, it suffices to locate the poles of the closed loop system in a sub region of the unit circle. But when, we are subject to a strong disturbance parameter ($\alpha>\alpha_{max}$) ,it can be observed that the output deviates more from the output of the unperturbed system. These results confirm our work.

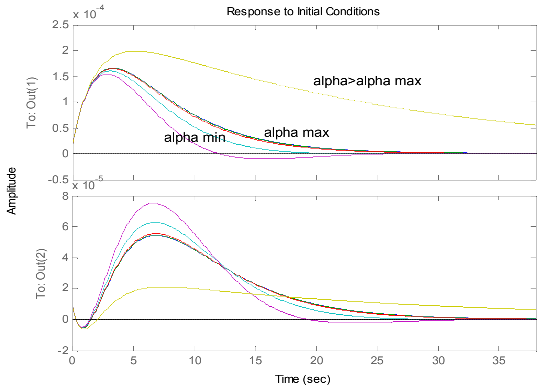

Fig.3. outputs of four compartments model with one perturbed parameter

5. Conclusion

In this paper we consider a discrete linear system with perturbed dynamic. We have proposed, under certain hypotheses, a control law which allows the insensitivity of the output to the disturbance. Future research will investigate the generalization of the developed technique for several uncertainties in the dynamic of the nominal system model.

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