Time-frequency Extraction Algorithm of Harmonic Signal from Lorenz Mixed Signal

Erfu Wang, Qiang Liu and Qun Ding

Key Laboratory of Electronics Engineering, College of Heilongjiang Province
Heilongjiang University, P. R. China,150080

Abstract

The separation of chaos and signal is an important problem of chaos signal processing. In recent years, the time-frequency analysis method is more and more mature. This paper first introduces the basic theory of time-frequency methods. We compare wavelet method with empirical mode decomposition (EMD) method, according to the different noise situation of the performance analysis of harmonic signal extraction in chaos background. Summarize a new synthesis about wavelet threshold and empirical mode decomposition (EMD) complementary of new harmonic signal extraction by experimental simulation. Computer simulation verified that the methods are high availability.

Keywords: *Harmonic Signal, Chaos, Wavelet, Empirical Mode Decomposition, Blind Sources Extraction.*

1. Introduction

Chaos is widespread in the various domains, such as chaos secure communication, ECM and heart computer signal processing in [1]. Chaos theory attracts a lot of attention by the scholars in the last decade.

Many researchers introduced various methods of detecting, separating and extracting signals according to the chaos' different characters. Leung uses smallest phase space capacity method to estimate polynomial parameter insert chaos [2]. With the help of the concept of differential manifold tangent space, Native Fuping Wang and some other ones use the inherent geometric properties of chaos attractor to realize the separation between chaos interference and weak signal in [3]. Haykin made a research on the extraction of small objectives signal from the ocean noise which is proved as chaos noise by the way of artificial nerve network in [4]. Short made a research on the signal extraction from chaos communication system in the way of chaos forecast method according to the chaotic signals characteristic of short-time forecast. These methods explored a new field of chaos signal processing, but lacked of systematicness. Some methods are rigorous and weak applicability, and demand target signal be much smaller than chaos background signal. In recent years, time-frequency analysis theory is more popular in [5], [6] and [7]. Time-frequency method [8], [9] and [10] can mainly carry out the problem of harmonic signal extraction from continuity chaos system background, achieve separation between chaos and signal according to different time-frequency character of chaos signal, noise signal and harmonic signal. So it can get useful signal from chaos background. When the signal-to-noise ratio (SNR) is not so weak, the extraction effect will be perfect.

Comparing wavelet method with empirical mode decomposition (EMD) method, according to the performance analysis of harmonic signal extraction from chaos background in different noise level and SNR, finally give the extraction procedure about complementary scheme. After theory analysis and a lot of simulation, the paper will give the corresponding results and analysis.

2. Chaotic System

We choose Lorenz chaotic system to simulate and experiment. Lorenz system is a three-dimensional continuous dynamic system, its nonlinear state equations are defined as follows:

$$\begin{cases} \dfrac{dx}{dt} = -a(x-y) \\ \dfrac{dy}{dt} = -xz + bx - y \\ \dfrac{dz}{dt} = xy - cz \end{cases} \tag{1}$$

In the following simulation experiment, we choose $a = 10$, $b = 28$ and $c = 8/3$ as the parameters. We get $x0 = y0 = z0 = 0.1$ and the step length is 0.01. We iterate 4000 points and use the 1900 to 4000 from x axis as the chaotic background sequence. Fig. 1 and Fig. 2 give the Lorenz phase diagrams, we can see that, when $a = 28$ the Lorenz system can evolve into chaotic state.

2.1 Basic Theory of Time-frequency Methods

Time-frequency methods are good at deal with non-stationary signal. It can extract the time-domain character and frequency-domain character at the meantime. Which is the most classical wavelet transform method, EMD as a new method of time-frequency signal processing in some application can get better than the wavelet transform.

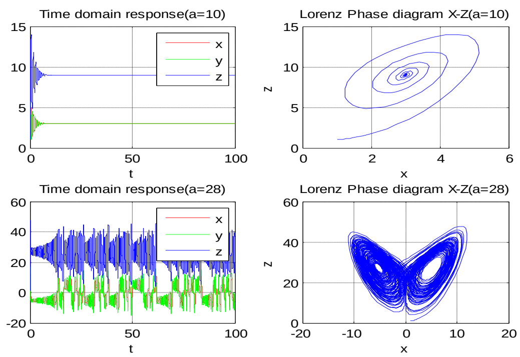


Fig. 1 The time domain response of Lorenz system and the phase diagram (a=10 and a=28).

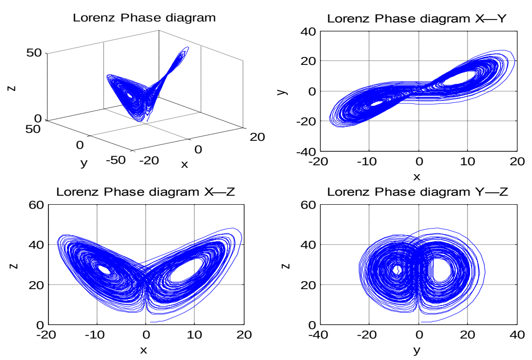


Fig. 2 The Lorenz Phase diagram (a=28).

Wavelet transform: Wavelet transform started to develop as a time-frequency Analysis method from the anaphase 20th century. To the present signal $x(t) \in L2(R)$, the continuous wavelet transform(CWT) of signal $x(t)$ is defined as:

$$WT_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi^*[\frac{t-b}{a}]dt$$
$$= \langle x(t), \psi_{a,b}(t) \rangle \qquad (2)$$

In the Eq.(2): a >0 is the scale factor and b is the shifted factor.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi[\frac{t-b}{a}] \qquad (3)$$

This equation is called wavelet primary function which is the shift of mother wavelet and scale dilation of the generating wavelet. Wavelet transform is a kind of correlated calculation between the originality signals and the group of wavelet functions after dilation essentially.

The analysis process based on wavelet transform is the decomposition and reconfiguration process virtually. The primary wavelet Daubechies is continuous, orthogonal and easy to implement. So the wavelet analysis part use the Daubechies. The db6 is chosen due to its good localized character and orthogonal character.

EMD method: EMD (Empirical Mode Decomposition) is a powerful tool to analyze non-linear and non-stationary signal and is instituted by N. E. Huang etc originally. EMD method is based on the in-depth study of the concept of instantaneous frequency, and it is closely related to the corresponding Hilbert transformation. Decompose non-linear and non-stationary signals can obtain a series IMF (Intrinsic Mode Function), which express the signal character and time scale. IMF is narrow band stationary signal. IMF must satisfy the following two conditions:

a) The discrepancy of zero-crossing and the extreme point is zero or one;

b) At any point, the mean of local maximum envelope and local minimum envelope is zero;

Only decompose the signal into some IMF, by analyzing every instantaneous frequency we can reveal true physics sense of original signal.

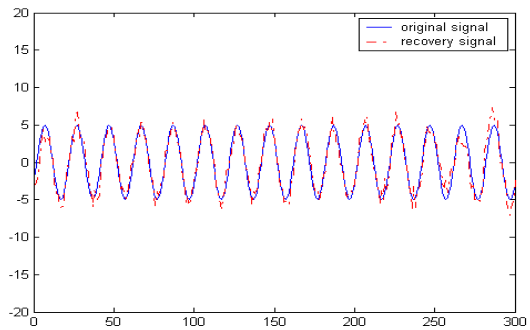
2.2 Wavelet threshold de-noising theory

Because of wavelet transform is linear, wavelet transformation coefficients are additive. When elect wavelet matching with signal to conduct wavelet transformation, signal energy mainly focus on a few sparse wavelet coefficients on some frequency band with relatively large amplitude. But wavelet transform of white noise is still white noise, which is widely distributed in every dimension of time axis and its amplitude is not big. So we can set a threshold, use the threshold to adjust wavelet coefficients according to certain rules. After adjustment, wavelet coefficients are reconstructed by using the inversion algorithm, we will get the target signal. This is the wavelet threshold de-noising theory basis. This paper choose heuristic threshold.

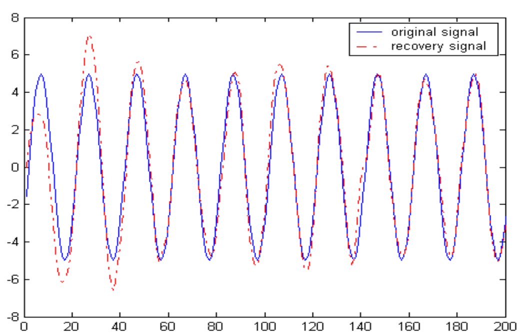
3. The Comparison of the Two Time-frequency Methods

3.1 The Influence of noise level (NL) on the Extraction Effect

The Lorenz system, Gauss noise and $s(n) = A\sin(2\pi f_n)$ mixed the composite signal, and $A = 5$, $f = 5Hz$. The noise level (NL) is that the standard ratio of Gauss noise to chaotic interference. Observe wavelet transform method and EMD method on harmonic signal extraction effect after changing NL.



(a) Wavelet method



(b) EMD method

Fig. 3 The recovery harmonic signal by wavelet method and EMD method when NL=10%.

From Fig. 3 we can see that two methods are perfect when NL=10%, there is definite noise in the chaotic background and NL is small.

When NL are 10%, 30%, 50%, 70%, 100%, signal is extracted from chaotic background with noise by using wavelet method and EMD method. Table1 gives quantitative comparison directly. R-wavelet means the correlation coefficient of recovery harmonic signal and original harmonic signal when use wavelet method. R-EMD means the correlation coefficient of recovery harmonic signal and original harmonic signal when use EMD method. The waveform almost distort by wavelet method when the noise is much bigger, then this method can't extract harmonic signal. EMD method is still useful and the effect is perfect. When NL=70%, the correlation coefficient is 0.5271 using wavelet method, and the correlation coefficient is 0.7286 when use EMD method, and the recovery level has a large improvement.

We can get a conclusion from Table1 by comparing the quantitative results of wavelet method and EMD method .When NL<30%, wavelet analysis method is useful and this extraction effect is perfect. When NL>50%, EMD method is better than wavelet method. The extraction effect is more perfect.

3.2 The Influence of SNR on the Extraction effect

The Lorenz system, Gauss noise and $s(n) = A\sin(2\pi f_n)$ mixed the composite signal, and fix $A = 5$, $f = 5Hz$. Fix NL=30% based on the influence of NL on the extraction performance. SNR means the energy ratio of harmonic signal to the sum of energy of chaotic interference and Gauss noise. Compare the extraction performance of wavelet method and EMD method after changing SNR.

When SNR are -1, -5, -10, -20, -30, we use wavelet method and EMD method as extraction method of harmonic signal from chaotic background with noise. Table1 directly gives the quantitative of correlation coefficient comparison.

We can get conclusions from Table 1 by comparing quantitative results of wavelet method and EMD method. When NL=30% and SNR>-5, wavelet method is better than EMD method. At this moment, the decomposition stability of wavelet plays a leading role When SNR<-5 or SNR is lower, EMD method is perfect. When NL is higher and SNR is low, the EMD method will be used to extract harmonic signal from chaotic background.

We can compare the two kinds of time-frequency extraction methods based on the simulation experiment and analysis mentioned above:

a) The performance of Wavelet Transform has better stability while EMD has worse stability. Because Wavelet decomposition completes all harmonics factorization in one operation while the process of EMD is Recurrence-by-component, if the previous component generates a false harmonic then the false harmonic will be progressively enlarged.

IJCSI International Journal of Computer Science Issues, Vol. 9, Issue 6, No 1, November 2012
ISSN (Online): 1694-0814
www.IJCSI.org

b) The Wavelet Transform method is better than the EMD especially to the signal with high noise level NL.

c) EMD method has higher accuracy than WT at the case of low SNR.

Table2 compares the Wavelet Transform with EMD from parameter set, convergence rate, stability and SNR. We can infer that WT and EMD are complementary in de-noising performance from the simulation result mentioned above.

Table 1: Extraction effect comparison in different NL and SNR
(a) Different NL

correlation coefficient	***NL***				
	10%	30%	50%	70%	100%
R-wavelet	0.9608	0.8370	0.6850	0.5271	0.4273
R-EMD	0.9451	0.8404	0.8285	0.7286	0.5545

(b) Different SNR

correlation coefficient	***SNR***				
	-1	-5	-10	-20	-30
R-wavelet	0.8949	0.7945	0.5907	0.2381	0.0271
R-EMD	0.8582	0.8398	0.7438	0.2589	0.1789

Table 2: De-noising performance of WT and EMD

effect	***NL***	
	Wavelet	***EMD***
Parameter set	first give wavelet primary function series	need not first set
Convergence rate	slow	fast
Stability	stable	unstable
SNR	higher	lower

Table 3: The correlation coefficient of mixed method after every step separation in the different NL

correlation coefficient	***NL***				
	10%	30%	50%	70%	100%
r-yz	0.9809	0.8500	0.6764	0.5665	0.4385
R-EMD	0.9772	0.7554	0.8241	0.7614	0.5692

Due to complementary characteristics of the Wavelet method and EMD in all aspects, it can switch between wavelet method and EMD method to choose better extracted performances of harmonic signal in the chaotic background, according to different noise level and SNR condition.

4. A New Method of Harmonic Signal Extraction

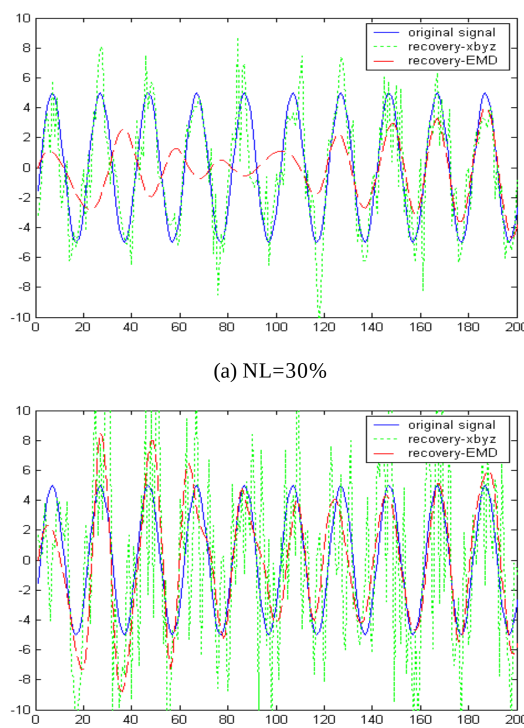
We will discuss a complementary and comprehensive method which combines wavelet transform, threshold de-noising and EMD method, according to complementary characteristics of the Wavelet method and EMD in the noise level and SNR aspect and two methods also can be used in de-noising speed and stability.

4.1 Experimental one

The Lorenz system, Gauss noise and $s(n) = A\sin(2\pi f_n)$ mixed the composite signal, and fix $A = 5$, $f = 5Hz$. Use wavelet method for mixed signal, then process it with EMD method, and observe the separation performance.

We respectively give the separation performance when NL=30% and NL=70%. "Original signal" is original harmonic signal, "recovery-xbyz" means the extraction signal after the wavelet threshold de-noising. "recovery-EMD" is recovery harmonic signal after the EMD method.

Table 3 is that the correlation coefficient of mixed method after every step separation in the different NL.



(a) NL=30%

(b) NL=70%

Fig. 4 The extraction effect of Wavelet-EMD method

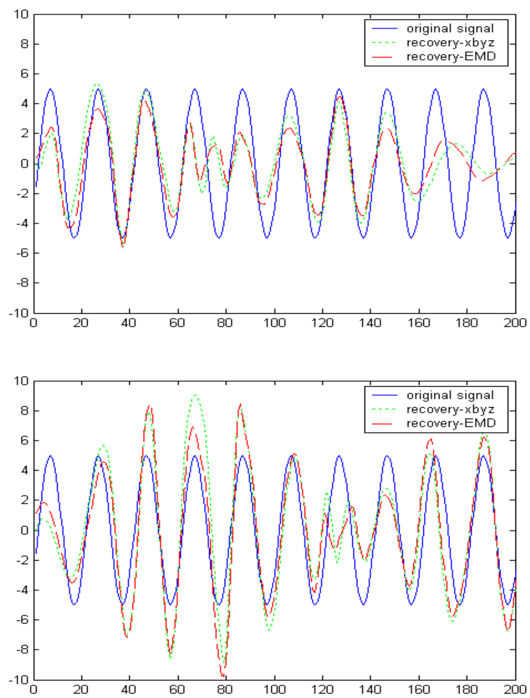


Fig. 5 The extraction effect of EMD-Wavelet method

From Fig. 4, we can know, using EMD method after reconstruction of wavelet threshold de-noising, which called as Wavelet-EMD method. When NL(<30%) is low in chaos background, the signal effect is perfect after reconstruction of wavelet threshold de-noising. Then using EMD method according to the change of correlation coefficient, we can see that the extraction effect become worse. This method is undesirable because that it not only increases the complexity of the algorithm but also reduces the result. The extraction waveform of Wavelet method is almost distorted when the noise is much bigger, then this method can't extract harmonic signal. The extraction effect is more perfect and has been improved largely after EMD method.

4.2 Experimental two

The Lorenz system, Gauss noise and $s(n) = A\sin(2\pi f_n)$ mixed the composite signal, and fix $A = 5$, $f = 5Hz$. We first use EMD method for the mixed signal, then use reconstruction of wavelet threshold method, and observe the separation performance at last.

From Figure. 5, there is an obvious improvement in the background of chaotic signal extracted effects in different levels of noise after processing it by EMD method and reconstruction wavelet threshold to remove noise, which called as EMD-Wavelet method. Because that EMD has advantage on fast convergent rate. If EMD works as the first step to remove noise, it can improve the signal-to-noise ratio and provide the appropriate conditions for application of wavelet transform. Then take advantage of good stability and high-precision for de-noising of wavelet as the second step, we can further improve extraction performance of harmonic signal.

5. Conclusion

This paper first introduces the basic theory of time-frequency methods. According to the different noise level and signal-to-noise ratio (SNR), we analyze the performance of harmonic signal extraction in the background of chaos, and compare wavelet method with empirical mode decomposition (EMD) method to find out both more applicable situation and its advantages. We summarize a synthesis method about wavelet threshold and empirical mode decomposition (EMD) complementary of new harmonic signal extraction by experimental simulation. Computer simulation verified that the methods are high availability. Finally give the extraction procedure about complementary scheme.

Acknowledgments

This work is supported by the National Science Foundation of China (no.60672011), the China Postdoctoral Science Foundation (no.2012M511525), the Key Laboratory of Electronics Engineering, College of Heilongjiang Province, Heilongjiang University Youth Fund (no. no.QL201025) and Dr. Start funds of Heilongjiang University.

References

[1] Richer M, Schreiber T and Kaplan D T, "Fetal EGG extraction with nonlinear state-space projection", IEEE Trans. Biom. Eng, vol. 45(1), pp.133-137, 1998.
[2] Leung Henry and Huang Xing-ping, "Sinusoidal frequency estimation in chaotic noise", ICASSP, vol. 2, pp. 1344-1347, 1995.
[3] Wang Fuping, Guo Jingbo, Wang Zanji and Xiao Dachuan , "Harmonic Signal Extraction from Strong Chaotic interference", Acta Physica Sinica, vol. 50(6), pp. 1019-1023, 2001.
[4] Haykin S and Li X B, "Detection of signal in chaos", Proceeding of IEEE, vol. 83(1), pp. 94-122, 1995.
[5] Mengmeng Zhang, Zeming Li, ChangNian Zhang, Huihui Bai, "Adaptive Feature Extraction and Image matching Based on Haar Wavelet Transform and SIFT", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 7, pp. 1 ~ 8, 2012

[6] Huang N E,Shen Z and Long S R, "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-station time series analysis", Proceeding of the Royal Society of London A, vol. 454, pp. 903-995, 1998.
[7] Saeed K. Amirgholipour, Ahmad R. Naghsh-Nilchi, "Robust Digital Image Watermarking Based on Joint DWT-DCT", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 3, No. 2, pp. 42 ~ 54, 2009
[8] Sun yankui, Wavelet analysis and its application, China Machine Press, China, 2005.
[9] M.R. Mirarab, H. Dehghani and Ali Pourmohammad, "A Novel Wavelet Based ICA Technique Using Kurtosis", The 2nd International Conference on Signal Processing Systems, pp. 36-39, 2010.
[10] Zhang Defeng, Matlab and Wavelet analysis, China Machine Press, China, 2010.

Erfu Wang Associate professor, Heilongjiang province, China, 1980. Received PhD degree and M.S. degree in Harbin Institute of Technology, China, in 2009 and 2005 respectively, and B.S. degree in Jilin University, China. The interest fields are blind source separation, array signal processing, wideband wireless communications, etc.

Qiang Liu Postgraduate student, Heilongjiang province, China, 1989. Received B.S. degree in Heilongjiang University, China. The interest fields are signal processing, blind source separation.

Qun Ding Professor, Heilongjiang province, China, 1957. Received PhD degree and M.S. degree in Harbin Institute of Technology, China, in 2007 and 1997 respectively. The interest fields are secure communication, information security, Pattern recognition, etc. Currently, she is a professor of Heilong Jiang University, China.