Study and Optimization of Cooperative Spectrum Sensing in OFDM Cognitive Radio Networks

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Abstract
In this paper, a cooperative sensing algorithm for OFDM is proposed in this paper based on the performance analysis of ONPD algorithm, then the system weight vector is optimized and the closed form expression of optimal weight vector is obtained using the maximum improved deflection coefficient. Theoretical analysis and simulation results demonstrate the remarkable improvement of proposed algorithm on detection performance compared with the classical cooperation detection algorithm.
Keywords: *Cyclic prefix; Cooperative sensing; Deflection coefficient; OFDM; Cognitive radio*

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has become the core modulation technique solution for next generation wireless communication. It has been used by primary users (PU) due to the character of multicarrier and high spectrum efficiency. Recently the study of detection of OFDM signal in cognitive radio (CR) system has drawn considerable attention. The detection algorithms of single cognitive user can be roughly divided into two categories: the algorithm based on Pilot Tones (PT) [1] and the algorithm based on Cyclic Prefix (CP) [2-5], for the reason that the PT mode is variable and difficult to obtain through estimation, more research is focus on the algorithm based on CP. The correlation of CP has been calculated to detect the PU signal in the time of a single OFDM block in [2], and then a new algorithm was proposed by extending the correlation detection to a series of consecutive OFDM blocks in [3]. After analysis of [2,6], the Optimal NEYMAN-PEARSON Detector (ONPD) was proposed based on NEYMAN-PEARSON criterion in [4]. And this ONPD algorithm does not need any prior knowledge about synchronization information with low complexity. Through related literature analysis, we found that most of OFDM signal detectors utilize frequency domain or time domain to help detection but pay little attention to the space resources. This may lead to the Hidden terminal problem and shadow fading problem. Utilizing space diversity technology, multi-user collaborative spectrum detection technology can solve hidden terminal problem and shadow fading problem of spectrum sensing in cognitive radio and improve the detection performance efficiency[7-10]. It is the original intention of the paper that how to combine the OFDM detector and the cooperative detection algorithm effectively in CR networks.
Based on the above considerations, we propose a new cooperation detection algorithm for OFDM signal. The contribution of this paper is as following: First, we proposed a new cooperative spectrum sensing algorithm for OFDM signal with soft combined decision fusion in cognitive radio networks; second, we utilize the maximum modified deflection coefficient to improve the detection performance and give the closed form expressions of optimal weighted vector.

2 System Model

2.1 Local Sensing Algorithm

The detector we proposed for single node of secondary network is based on the ONPD derived in [4]. Utilizing the feature of OFDM signal, ONPD is suitable for multi-CR detection in cognitive networks with low complexity. Each CR node samples signal in the same interested frequency band to detect the OFDM signal of PU during the sensing period. We assume that each OFDM block obtains data with length of N_d and CP with length of N_c. Without loss of generality, we sample from $K+1$ OFDM blocks in order to ensure the total number of K OFDM blocks. The received vector x consists of K consecutive OFDM symbols. After defining the sample value product $r_i \triangleq x_i x_{i+N_d}^*$, we have $R_i \triangleq \frac{1}{K}\sum_{k=0}^{K-1} r_{i+k(N_c+N_d)}$, $i=0,\cdots,N_c+N_d-1$. When PU is present, there will be N_c consecutive values of R_i that have a different distribution from the other N_d values, otherwise all the averaged sample value products R_i $(0 \le i \le N_c+N_d-1)$ are identically distributed. Given the synchronization mismatch τ, S_τ denotes the set of consecutive indices, $\Re(\bullet)$ denotes taking the real part. The ONPD detector is given as follow:

$$C=\sum_{i\in S_\tau} \Re\ R_i \mathop{\gtrless}_{H_0}^{H_1} \lambda \qquad (1)$$

The distribution of $R_{n,i}$ has been given by [4], but some important parameter we need has not been given. Due to the mutual independence of $R_{n,i}\ i\ \ i\in S_\tau$, we can obtain the distribution of the detection result $C_i=\sum_{i\in S_\tau} \Re\ R_{n,i}$ of the ith CR is as follows

$$C_i \sim \begin{cases} N\left(0,\dfrac{N_c\sigma_{n,i}^4}{2K}\right) & H_0 \\ N\left(N_c h_i^2\sigma_s^2, N_c\dfrac{h_i^4\sigma_s^4+\sigma_{n,i}^4/2+h_i^2\sigma_s^2\sigma_{n,i}^2}{K}\right) & H_1 \end{cases} \qquad (2)$$

From the distribution ,we can get the expectation and variance of C_i in the two kinds of test conditions H_0 and H_1 respectively.

$$\begin{cases} \mu_{0,i}=E\ C_i|H_0\ =0 \\ \mu_{1,i}=E\ C_i|H_1\ =N_c h_i^2\sigma_s^2 \end{cases} \qquad (3)$$

$$\begin{cases} \xi_{0,i}=\text{var}\ C_i|H_0\ =\dfrac{N_c\sigma_{n,i}^4}{2K} \\ \xi_{1,i}=\text{var}\ C_i|H_1\ =N_c\dfrac{h_i^4\sigma_s^4+\sigma_{n,i}^4/2+h_i^2\sigma_s^2\sigma_{n,i}^2}{K} \end{cases} \qquad (4)$$

2.2 OFDM Cooperative Sensing Model

The sensing system of cognitive networks is shown in figure 1, N CR nodes detect PU signal and use the linear weighting mode in fusion center. Sensing process is divided into sensing stage and reporting stage. After sensing stage the received signal in CR is given as follow:

$$\mathbf{x}_i=\begin{cases} \mathbf{n}_i & H_0 \\ h_i\mathbf{s}+\mathbf{n}_i & H_1 \end{cases} \quad i=1,2,\cdots N \qquad (5)$$

Where $\mathbf{x}_i$ denotes the received signal vector by the ith CR node, and h_i denotes the attenuation factor of the sensing channel from CR_i to PU, $\mathbf{s}$ denotes a sequence of K consecutively transmitted OFDM symbols, and $\mathbf{n}_i$ is a noise vector, and $\mathbf{n}_i \sim N_c(0,\sigma_{n,i}^2\mathbf{I}_N)$ where $\sigma_{n,i}^2$ is the noise variance of the ith sensing channel and $N_c\ \bullet$ represents the complex Gaussian distribution.

Each CR use the ONPD algorithm to detect the receive signal samples during the sensing stage, and then the accumulative results of ONPD are reported to the FU through the Completely orthogonal report channels. The information FU received from the ith CR is $y_i=C_i+v_i$, then FU uses the linear weighted method to combine the data from all the CRs, and judges whether there is PU signal or not after comparing with a predefined Threshold.

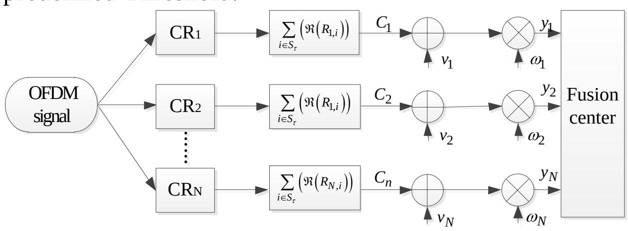

Fig 1 the OFDM cooperative sensing algorithm structure

FU receives the detection data C_i from different CRs through the report channels and fuses these data in order to improve the detection efficiency. according to the Fig 1 and type (3), the received data y_i by FU obey the Gaussian distribution, their expectations are the same as C_i for the reason that $E\ y_i\ =E\ C_i$ and variances can be obtained as

$$\begin{cases} \zeta_{0,i}=\text{var}\ y_i|H_0\ =\dfrac{N_c\sigma_{n,i}^4}{2K}+\sigma_{v,i}^2 \\ \zeta_{1,i}=\text{var}\ y_i|H_1\ =N_c\dfrac{h_i^4\sigma_s^4+\sigma_{n,i}^4/2+h_i^2\sigma_s^2\sigma_{n,i}^2}{K}+\sigma_{v,i}^2 \end{cases} \qquad (6)$$

After receiving y_i , the FU judges whether the PU signal is present or not through linear fusion as follow

$$y_c=\sum_{i=1}^{N}\omega_i y_i=\boldsymbol{\omega}^T\mathbf{y} \mathop{\gtrless}_{H_0}^{H_1} \gamma \qquad (7)$$

Where γ is threshold of the global judgment, and the $\boldsymbol{\omega}$ is the weight vector $\boldsymbol{\omega} \triangleq \omega_1,\omega_2,\cdots,\omega_N{}^T$. $\boldsymbol{\omega}$ can directly reflect the size of CR's contribution to the fusion result, which relates to the state of the sensing channel and report channel. According to the formula (4) (5) (6) and (7), we can infer that y_c follows the Gaussian distribution, the expectation and variance of H_0 and

H_1 is given as respectively

$$\begin{cases} E\left(y_c \middle| H_0\right) = 0 \\ E\left(y_c \middle| H_1\right) = \boldsymbol{\omega}^T \boldsymbol{\mu}_1 \end{cases} \quad (8)$$

$$\begin{cases} \mathrm{var}\left(y_c \middle| H_0\right) = \boldsymbol{\omega}^T \boldsymbol{\Sigma}_0 \boldsymbol{\omega} \\ \mathrm{var}\left(y_c \middle| H_1\right) = \boldsymbol{\omega}^T \boldsymbol{\Sigma}_1 \boldsymbol{\omega} \end{cases} \quad (9)$$

Where $\boldsymbol{\mu}_1$ is N-dimensional column vector, $\boldsymbol{\mu}_1 = \left(N_c h_1^2 \sigma_s^2, \cdots, N_c h_N^2 \sigma_s^2\right)^T$, both $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}_1$ is $N \times N$ diagonal matrix, $\boldsymbol{\Sigma}_0 = diag\left(\zeta_{0,1}, \cdots, \zeta_{0,N}\right)$, $\boldsymbol{\Sigma}_1 = diag\left(\zeta_{1,1}, \cdots, \zeta_{1,N}\right)$.

The false alarm probability and detection probability of the whole cooperative algorithm were given respectively as follow.

$$P_f = Q\left(\frac{\gamma}{\sqrt{\boldsymbol{\omega}^T \boldsymbol{\Sigma}_0 \boldsymbol{\omega}}}\right) \quad (10)$$

$$P_d = Q\left(\frac{\gamma - \boldsymbol{\omega}^T \boldsymbol{\mu}_1}{\sqrt{\boldsymbol{\omega}^T \boldsymbol{\Sigma}_1 \boldsymbol{\omega}}}\right)$$

3 Optimization Algorithm Based on the Improved Deflection Coefficient

As the single parameter to reflect the performance of binary detection algorithm, the deflection coefficient is different from the false alarm probability and detection probability, since the two parameters can reflect detection performance only when they are considered jointly.

In general, the bigger value of deflection coefficient means the better detection performance. Conversely, the smaller value of deflection coefficient, the relatively poorer performance of the detection algorithm [11]. We use maximization deflection coefficient method to optimize the weighted vector of global judgment. The calculation formula of the deflection coefficient is of two kinds[11]: the denominator is $\mathrm{var}\left(y_c \middle| H_0\right)$ and $\mathrm{var}\left(y_c \middle| H_1\right)$ respectively. We propose the improved deflection coefficient combining two kinds of denominator in this paper, and the denominator adopts $p\,\mathrm{var}\left(y_c \middle| H_1\right) + (1-p)\,\mathrm{var}\left(y_c \middle| H_1\right)$ in the paper. The optimization goal is to maximize the improved deflection coefficient $d^2(\boldsymbol{\omega})$ through choosing the right $\boldsymbol{\omega}$. We can obtain the mathematical model as follow.

$$\begin{aligned} \underset{\boldsymbol{\omega}\geq 0, \|\boldsymbol{\omega}\|=1}{\arg\max}\, d^2(\boldsymbol{\omega}) &= \frac{\left[E\left(y_c \middle| H_1\right) - E\left(y_c \middle| H_0\right)\right]^2}{p\,\mathrm{var}\left(y_c \middle| H_1\right) + (1-p)\,\mathrm{var}\left(y_c \middle| H_1\right)} \\ &= \frac{\left(\boldsymbol{\omega}^T \boldsymbol{\mu}_1\right)^2}{p\left(\boldsymbol{\omega}^T \boldsymbol{\Sigma}_1 \boldsymbol{\omega}\right) + (1-p)\left(\boldsymbol{\omega}^T \boldsymbol{\Sigma}_0 \boldsymbol{\omega}\right)} \qquad 0 \leq p \leq 1 \end{aligned} \quad (10)$$

The vector $\boldsymbol{\omega}$ is the only variable in the model with $\|\boldsymbol{\omega}\| = 1$, and $d^2(\boldsymbol{\omega})$ is the continuous functions of $\boldsymbol{\omega}$, it is not difficult to find that $d^2(\boldsymbol{\omega})$ has a maximum value when $\boldsymbol{\omega}$ satisfies the condition of $\frac{\partial d^2(\boldsymbol{\omega})}{\partial \boldsymbol{\omega}} = 0$, we can write it as follow.

$$\begin{aligned} \frac{\partial d^2(\boldsymbol{\omega})}{\partial \boldsymbol{\omega}} &= \frac{2\boldsymbol{\mu}_1 \boldsymbol{\mu}_1^T \left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]\boldsymbol{\omega}}{\left\{\boldsymbol{\omega}^T \left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]\boldsymbol{\omega}\right\}^2} \\ &- \frac{2\left(\boldsymbol{\omega}^T \boldsymbol{\mu}_1\right)^2 \left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]\boldsymbol{\omega}}{\left\{\boldsymbol{\omega}^T \left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]\boldsymbol{\omega}\right\}^2} = 0 \end{aligned} \quad (11)$$

The expression of weighted vector $\boldsymbol{\omega}_{opt}$ satisfying (11) can be calculated as follow

$$\boldsymbol{\omega}_{opt} = \frac{\boldsymbol{\omega}^T \left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]\boldsymbol{\omega}}{\boldsymbol{\omega}^T \boldsymbol{\mu}_1} \left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]^{-1} \boldsymbol{\mu}_1 \quad (12)$$

The value of $\frac{\boldsymbol{\omega}^T \left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]\boldsymbol{\omega}}{\boldsymbol{\omega}^T \boldsymbol{\mu}_1}$ is a positive real number, and it doesn't affect the value of $\boldsymbol{\omega}_{opt}$ which will be normalization at last, so after normalization $\boldsymbol{\omega}_{opt}$ can be written as follow

$$\boldsymbol{\omega}_{opt} = \varepsilon\left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]^{-1} \boldsymbol{\mu}_1 \quad (13)$$

Where $\varepsilon = \frac{1}{\left\|\left[p\boldsymbol{\Sigma}_1 + (1-p)\boldsymbol{\Sigma}_0\right]^{-1} \boldsymbol{\mu}_1\right\|}$.

We can find $\boldsymbol{\omega}_{opt}$ can be easy to obtain from formula (13), and we analysis the impact of p value on the performance of cooperative detection in Fig 3.

4 Simulation Results

In this section computer simulation and analysis is proposed in order to verify the correctness and effectiveness of the proposed algorithm. All the simulations are obtained by Monte Carlo method, the sensing channels are set to be Gaussian fading. The SNR of the sensing channels are generated randomly with a average SNR. we set the FU receive information from CR_i without fading and the simulation model are designed as Fig 1.

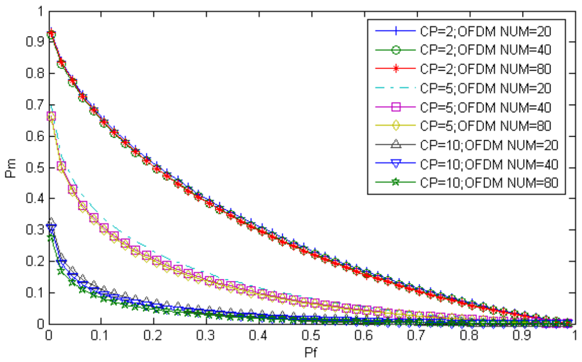

Fig 2 The ROC curves of the optimization algorithm with various CP number and various OFDM block number

Fig 2 shows the comparison of the simulation performance when the CP number and OFDM block number are different. the CR number is $N=5$, and the average SNR is $\overline{SNR}=-10dB$ in the simulation. the CP number of PU OFDM signal N_c is set to 2, 5, 10 respectively and the OFDM number of PU signal K is set to 20, 40, 80 respectively. From Fig 2, it can be seen that the bigger the CP number and OFDM number of PU are set , the better the performance of algorithm can be obtained. It is remarkable that the CP number seems to play the bigger role than OFDM number in the detection algorithm.

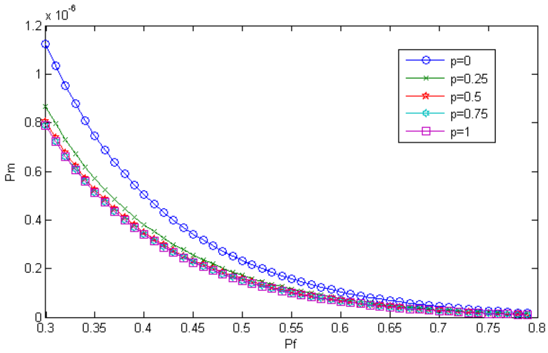

Fig 3 The ROC curves of the optimization algorithm with various values of *p*

The different value of *p* in formula (3) will lead to the different value of weight vector $\boldsymbol{\omega}$ and then will cause the various detection performance. Fig 3 shows the detection performance caused by the different value of $p(0 \le p \le 1)$. we set $\overline{SNR}=-2dB$, $K=5$, $N=5$, $N_c=5$ in this simulation. From Fig 3, it can be found that the detection performance of the proposed algorithm can be improved nonlinearly with the increase of the value of *p*, and the detection performance is optimum when *p*=1.

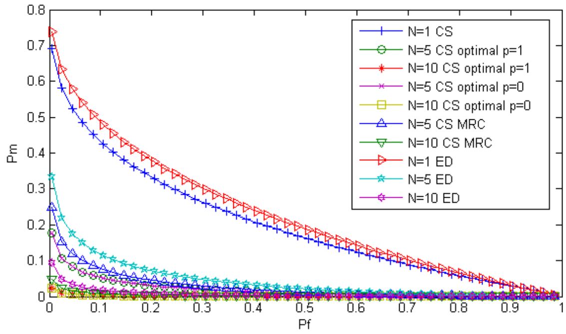

Fig 4 the ROC curves of optimization algorithm, MRC algorithm and energy detection algorithm

Fig 4 shows that the ROC curves of the optimization algorithm of ONPD, Maximal-Ratio Combing (MRC) of ONPD and the cooperation energy detection algorithm with various CR number. we set $\overline{SNR}=-2dB$, $K=5$, $N_c=5$ in this simulation, and the cooperation energy detection algorithm also uses MRC fusion algorithm in FU. The simulation results in Fig 4 shows that the detection performance of the proposed optimization algorithm is better than the MRC collaborative detection of ONPD and collaborative energy detection, and the bigger the CR number is, the better the detection performance is. It's important to note that when average SNR is small enough, the performance differences between $p=0$ and $p=1$ are tiny, and the detection performance curves of $p=0$ and $p=1$ coincide with each other.

Conclusions

In the paper, the detection performance of the ONPD algorithm is analyzed and a multiuser cooperation detection algorithm for OFDM signal is proposed, and the performance parameters of the algorithm have been calculated. Furthermore, we have derived the close form of optimal weight vector using the maximum improved deflection coefficient. The simulation results show that the proposed algorithm is better than other cooperative detection algorithm with good performance.

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