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A New Signal Denoising Method using Iterative Thresholding of the Spectral Intrinsic Decomposition

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Abstract − This paper presents a new signal denoising method based on the classical three step procedure analysis-threshold-synthesis and the Spectral Intrinsic Decomposition (SID). This method consists of an iterative thresholding of the SID components. If the wavelets denoising approach depends on the choice of the wavelet form, the SID-denoising proposed in this paper is self adaptive. The SID-based removal method reduces noise and can retain useful discontinuities of the signal as effectively as the wavelet techniques based on soft thresholding.

keywords
Spectral Intrinsic Decomposition, Signal denoising, Shrinkage, Soft thresholding, Wavelets.

1 Introduction

The Spectral Intrinsic Decomposition Method [1] is an adaptive decomposition technique with which any complicated signal can be decomposed into a definite number of high frequency and low frequency components called Spectral Proper Mode Functions (SPMFs). The decomposition procedure is adaptive and data-driven. The SPMFs are stationary and suitable for signal analysis. Assume that an observed data $s(t) = y(t) + n(t)$, contains the true signal $y(t)$ with additive noise $n(t)$ as function in time t to be sampled. Some time series denoising algorithms like wavelet transform model are widely used to deal with noise within the data observations. However, for non-linear and non-stationary time series, wavelet approaches can fail. In the L-level wavelet decomposition of a signal, the number of coefficients with significant energy is small. This is a direct consequence of the approximation property of the wavelets. The signal can be accurately represented by a small number of coefficients. Wavelet shrinkage, developed by Johnstone and Donoho [2], selects these coefficients by thresholding. In the same spirit of wavelet denoising approach and following the work published in [4], where the authors show how Empirical Mode Decomposition (EMD) [3] reveals an equivalent filter bank structure which shares most properties of a wavelet decomposition, O.Niang in [5] and K. Khaldi and al. in [6] proposed EMD-based shrinkage method for signal denoising. In the same vein, this paper introduces and tests a new approach based on SID thresholding for signal denoising. In section 2, we describe the Spectral Intrinsic Decomposition principle and recall the wavelets decomposition principle. Section 3 concerns some wavelets thresholding denoising methods. Section 4 exposes the SID-based denoising method. Some test results are presented in section 5, and we finish by conclusions and perspectives to this work in section 6.

2 The Spectral Intrinsic Decomposition

SID method decomposes a complex signal (e.g. a signal with coexisting several characteristic time scales) into elementaries AM-FM type components, called Spectral Proper Mode Functions (SPMFs). The Spectral Intrinsic Decomposition express an non-linear signal into a linear combination of the eigenvectors of all the PDE- envelope operator defined in [1]. Let us denote by $\mathbf{E}$ (see [1]) the envelope of any one-dimensional discrete signal $\mathbf{S}$, the eigen decomposition of $\mathbf{E}$ gives : $[\mathbf{V_E}, \mathbf{L_E}] = eig(\mathbf{E})$, where $\mathbf{V_E} = [\mathbf{V_1}, \ldots, \mathbf{V_{size(S)}}]$ and $\mathbf{L_E} = [\mathbf{L_1}, \ldots, \mathbf{L_{size(S)}}]$. The reconstruction coefficient of $\mathbf{S}$ is given by : $\mathbf{C} = \mathbf{L_E}\mathbf{V_E}^{-1}\mathbf{S}^{\perp}$.

Algorithm 1 : Spectral Intrinsic Decomposition [1]

1: **compute the diffusivity function** $g^{\pm}$ from $\mathbf{S}_0$,
2: **compute** matrix operator $\mathbf{L}^{-1} = \mathbf{E}$
3: **perform** eigen decomposition of $\mathbf{E}$, $[\mathbf{V_E}, \mathbf{L_E}] = eig(\mathbf{E})$.
4: **perform** Reconstrution Coefficients of $\mathbf{S}_0$, $\mathbf{C} = \mathbf{L_E}\mathbf{V_E}^{-1}\mathbf{S}_0^{-1}$.
5: **set** $[\mathbf{V_k}]$, and $[\mathbf{L_k}]$ for $k = 1 \ldots N$, ▷ *Result* and $\mathbf{S_0} \leftarrow \sum_{k=1}^{N} \mathbf{V_k} * \mathbf{C_k}$

So, SID of $\mathbf{S}$ as defined in algorithm 1, can finally be presented with the following representation

$$\mathbf{S} = \sum_{k \in \{j/\lambda_j = 1\}} \mathbf{V_k}\mathbf{C}_k + \sum_{k \notin \{j/\lambda_j = 1\}} \mathbf{V_k}\mathbf{C}_k. \quad (1)$$

Hence

$$\mathbf{S} = \sum_{k=1}^{N} \mathbf{SPMF_k},$$

where $SPMF_k = \mathbf{V_k}\mathbf{C}_k$ is the $k-$th spectral proper mode(or SPMF) of the signal. In all cases, an SPMFs can be viewed as a (nonlinear) frequency narrow-band wavelet φ with Amplitude Modulation by a lower frequency signal $A[n]$

$$SPMF_k[n] = A_k[n]\ \varphi_k[n].$$

In stochastic situations involving broadband noise, one can make an interpretation of SID in terms of a constant$-Q$ filter bank [8, 7] and as a data-driven wavelet-like expansion [4] or as an sparse representation of non linear signal [1].

3 Some wavelet Denoising methods

Denoising by thresholding in the wavelet domain has been developed principally by Donoho et al. in [2, 9, 10, 11].

Wavelet transforms express the signal in terms of wavelet coefficients, describing the signal variation at different scales. The discrete wavelet transform represents a one-dimensional signal s into shifted versions of a dilated low-pass scaling function φ, and shifted and dilated versions of a bandpass wavelet function ψ. In case of orthonormal wavelets, we have

$$s = \sum_{i \in \mathbb{Z}} \left\langle s, \varphi_i^J \right\rangle \varphi_i^J + \sum_{j=-\infty}^{J} \sum_{i \in \mathbb{Z}} \left\langle s, \psi_i^j \right\rangle \psi_i^j,$$

where the lower index i stands for spatial position, upper index j represents the level of scale, up to a chosen maximum J, and where $\psi_i^j(t) = 2^{-j/2}\psi\left(2^{-j}t - i\right)$, and with $\langle \cdot, \cdot \rangle$ denoting the inner product in $L_2(\mathbb{R})$.

If the wavelet basis is chosen properly, a signal will be generally described by only a few significant wavelet coefficients, while moderate white Gaussian noise pollutes all the wavelet coefficients by a small amount. Signal denoising by wavelet shrinkage starts from this assumption, and creates a smoothed version of the processed signal by the following three-step procedure analysis-shrinkage-synthesis.

Various shrinkage functions leading to qualitatively different denoised functions $\hat{s}$ were considered in literature, e.g. linear shrinkage, and nonlinear shrinkage functions such as soft, garrote, firm and hard shrinkage. For example the soft wavelets shrinkage function

$$S_\theta(x) = \operatorname{sgn}(x)\left(|x| - \theta\right)_+,$$

where the threshold θ can be estimated by the following expression [2, 9, 10, 11]

$$\theta = \sqrt{2\ \log(N)}\,\sigma,$$

where N is the signal length, and where σ is the noise level (standard deviation). The shrinkage parameter θ is chosen with respect to the amount of noise in the input signal. In general, the denoised solution $\hat{s}$ is obtained from s using a single step of this multiscale procedure, e.g. the method is applied noniteratively and is known as the classical Multiple Level Single Iteration (MLSI) scheme. Other schemes exist where the method is applied iteratively [12], like Single Level Iterated (SLI) and Multiple level Iterated (MLI). When the noise level σ is unknown, an estimation can be proceed via the median absolute deviation of the wavelet coefficients at the finest scale of resolution, j=1, such that $\hat{\sigma}$=$\hat{\sigma}_1$=1.4826 median($|d_i^1|$). According to [13], different threshold θ_j are used at each level j according to the rule

$$\theta_j = \theta_1/\sqrt{2}^{j-1}, \quad (2)$$

where $\theta_1 = \sqrt{2\log(N)}\sigma_1$. This choice leads to significantly reduced oscillations (Gibbs phenomenon) near discontinuities of the reconstructed signal.

4 Iterative SID Denoising Method

Following the above wavelet shrinkage method, one can adopt a similar process in order to suppress small fluctuations in the SPMFs resulting from Spectral Intrinsic Decomposition of a noisy signal. When wavelet shrinkage depends on the choice of wavelet basis or mother wavelet, the SID is adaptive and generally, can gives most of the dynamic of a noisy signal by the SPMFs corresponding to higher eigenvalue (nearest to 1). Signal denoising by iterative SID threshold principle comes from this assumption, and creates a smoothed version of the processed data by the following three-step procedure, analysis-threshold-synthesis. The iterative SID thresholding method for noise removal is described in algorithm 2. First we compute the SID decomposition of an noisy signal S, and choose all the $SPMF$ components corresponding to eigenvalues equal to 1 that gives the first scale of the restored input signal e.g

Algorithm 2 : Denoising by iterative SID Thresholding

Choose p, the filtering level , SRN_c and ϵ as a target precision ▷ *Initialization*

$m = 1, \hat{S}_m = S, N =$ length (N)

repeat

Decompose the noisy data S into the SPMFs ▷ *Analysis* $SPMF_k$ $(k = 1, \ldots, N)$.

$$\{SPMF_k\} \leftarrow SID(S)$$

Compute denoised signal at level m $\hat{S}_m$ of S ▷ *Synthesis*

$$\hat{S}_m \leftarrow \sum_{k\in\{j/\lambda_j=1\}} \mathbf{V_k C}_k$$

Compute the SRN

Add to S_m **the SPMFs** corresponding to the median third values ▷ *Enrichment* of $L_{med} = \{k/\lambda_k = 1\}$ in the SID decomposition.

$S = \hat{S}_m + \sum_{k\in L_{med}} \mathbf{V_k C}_k$

$m = m + 1$

until $m = p$ **or** $SRN - SNR_c \le \epsilon$

$\hat{S}_1$. After, for the second level denoising process, we enrich S_1 by considering all the third median eigenvalues different to 1, e.g L_{med} and recompose now the new input signal

$$S_{p=1} = \sum_{k\in\{j/\lambda_j=1\}} \mathbf{V_k C}_k + \sum_{k\in L_{med}} \mathbf{V_k C}_k. \quad (3)$$

This procedure can be repeated until the level $p = p+1$. If we need more filtering, we can control the estimation of the Signal-to-Noise-Ratio SNR. For this, we consider $\hat{S}_p$ as the new input signal, and we repeat the same process as described in algorithm 2. Thus we obtain a multi-scale denoising process with a sequence $(\hat{S}_p)$ of filtered versions of the noisy signal $S_0 = S$. The choice of L_{med} is empiric but can be optimized by regularization [14].

5 Results

Standard synthetic signals were corrupted with additive Gaussian noises, and then denoised by the proposed iterative thresholding SID approach. Performance of the denoising procedures was studied as a function of the signal-to-noise ratio (SNR) of the initial signal. The SNR was used as the objective figure of merit for comparing the original signal $s[n]$ with the denoised estimate $\hat{s}[n]$, and computed as

$$SNR = 10\log_{10}\frac{\sum_n |s\,[n]|^2}{\sum_n |s\,[n] - \hat{s}\,[n]|^2},$$

—	PSRN iterat 1	PSRN iterat 2	Numb of iterat
Blocks	23.4662	20.0259	2
Wave form	23.8233	11.1854	2
Heavy-sine	28.4642	29.7222	2

TABLE 1 – The associated signal-to-noise ratio (SNR) in dB for each iteration.

To test the applicability of the proposed desoining approach, we perform experiments on input signals `Blocks`, `wave form` and `Heavy-Sine`, which are of the standard signals in wavelet denoising (see WaveLab package in [15]).

The signals, of length $L = 2048$ points, and theirs noised versions are shown in Fig. 1,2 and 3. In tests, SID-based desoising (Algorithm 2) uses the threshold L_{med} which can be estimated by an Tykhonov regularization applied to the sequence of all the eigenvalues. Maximum levels of the multisclae SID denoising method in our tests is fixed to $p = 3$. In Figure 5, we summarize the parameters of our tests, the Signal-to-Noise-Ratio is improved with two iterations. In Fig 1 and 2, the first iteration gives better results while the second iteration $p = 2$ leads a loss of data. In Fig 3 the same work is performed and we can see that SID-based denoising method gives a gain from approximately 5 to 9 dB at the iteration $p = 2$, see Table 5 Generally at $p = 3$, only the SPMFs associated to 1 are retained. For $p > 3$, the process leads to a significant loss of information.

6 Conclusion

We have shown that the denoising approach consisting to iterative soft thresholding in SID components is suitable for noise removal. SID denoising method is adaptive contrary to wavelet methods. The results in this paper can be extended in several directions. One can study iterated single- or multi-level SID shrinkage to make extensive comparisons with iterated single- or multi-scale wavelet shrinkage. In our ongoing work, we will also consider the two-dimensional case.

Références

[1] Oumar Niang, Abdoulaye Thioune Eric Delechelle and Jacques Lemoine, " Spectral Intrinsic Decomposition Method for Adaptive Signal Representation", published in ISRN Signal Processing October 2012.

[2] D.L. Donoho and I.M. Johnstone, "Ideal spatial adaption via wavelet shrinkage," Biometrika, vol. 81(3), pp. 425-455, 1994.

[3] N.E. Huang, Z. Shen, S.R. Long, M.L. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung, and H.H. Liu, "The empirical mode decomposition and Hilbert spectrum

for nonlinear and non-stationary time series analysis," Proc. Roy. Soc. London A, vol. 545, pp. 903-995, 1998.

[4] P. Flandrin, and P. Gonçalvès, "Empirical Mode Decompositions as a Data-Driven Wavelet-Like Expansions," Int. J. of Wavelets, Multires. and Info. Proc., vol. 2(4), pp. 477-496, 2004.

[5] O. Niang, "Empirical Mode Decomposition : Contribution 'a la modélisation mathématique et application en traitement du signal et l'image," PhD thesis, Univ Paris 12, Créteil, France, Septembre 2007.

[6] Kais Khaldi, Abdel-Ouahab Boudraa, Abdelkhalek Bouchikhi, and Monia Turki-Hadj Alouane, "Speech Enhancement via EMD", EURASIP Journal on Advances in Signal Processing Volume 2008, Article ID 873204.

[7] P. Flandrin, G. Rilling, and P. Gonçalvès, "Empirical mode decomposition as a filter bank," IEEE Signal Processing Letters, vol. 11, pp. 112-114, 2004.

[8] Z. Wu and N.E. Huang, "A study of the characteristics of white noise using the Empirical Mode Decomposition method," Proc. Roy. Soc. London A, vol. 460, no. 2046, pp. 1597-1611, 2004.

[9] D.L. Donoho and I.M. Johnstone, "Adapting to unknown smoothness via wavelet shrinkage," Journal of the American Statistical Association, vol. 90(432), pp. 1200-1224, 1995.

[10] D.L. Donoho, "Denoising via soft thresholding, IEEE Transactions on Information Theory," vol. 41, pp. 613-627, 1995.

[11] D.L. Donoho, I.M. Johnstone, G. Kerkyacharian, and D. Picard, "Wavelet shrinkage : Asymptopia ? with discussion," Journal of the Royal Statistical Society, Series B, vol. 57(2), pp. 301-369, 1995.

[12] P. Mrázek, J. Weickert, G. Steidl and M. Welk, "On Iterations and Scales of Nonlinear Filters," O. Drbohlav (ed.) : Computer Vision Winter Workshop 2003, Valtice, Czech Republic, pp. 61-66. Czech Pattern Recognition Society, 2003.

[13] G. Steidl, J. Weickert, T. Brox, P. Mrázek, and M. Welk, "On the equivalence of soft wavelet shrinkage, total variation diffusion, total variation regularization, and SIDEs," Technical report, Series SPP-1114, Department of Mathematics, University of Bremen, Germany, 2003.

[14] O. Niang,E. Deléchelle and J. Lemoine," An Spectral approach for sifting process in Empirical Mode Decomposition," IEEE Transaction on Signal Processing, vol.58, num 11, pp. 5612-5623, 2010.

[15] Wavelab, developed by Buckheit, Chen, Donoho, Johnstone & Scargle (1995) at Stanford University, url : $http://www-stat.stanford.edu/\sim wavelab.$

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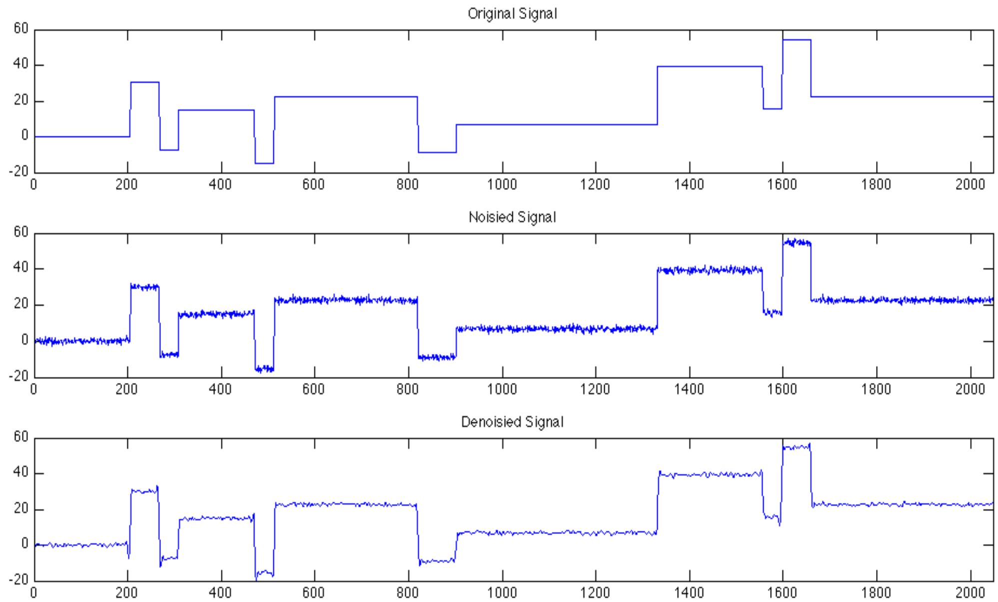

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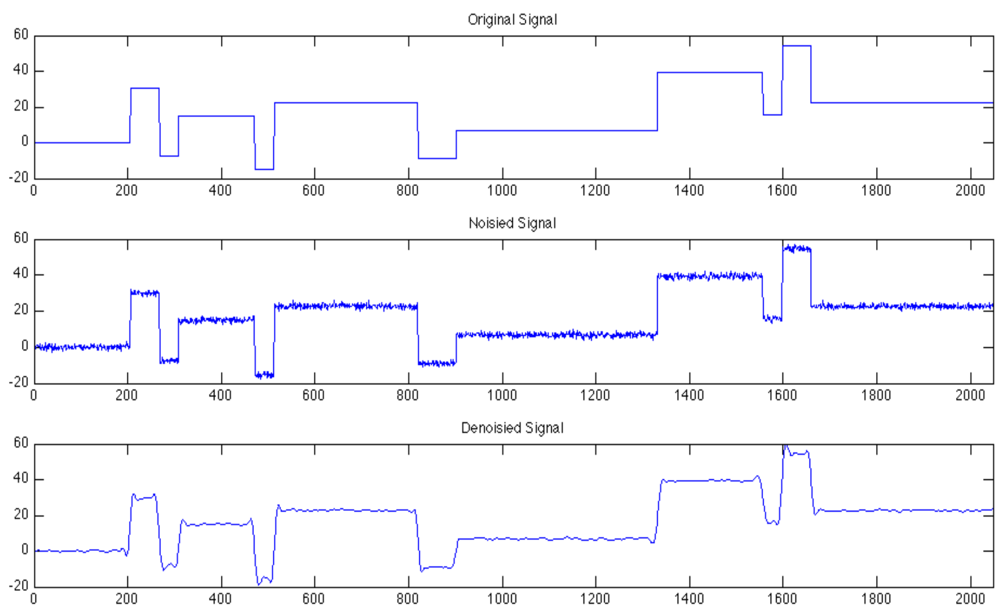
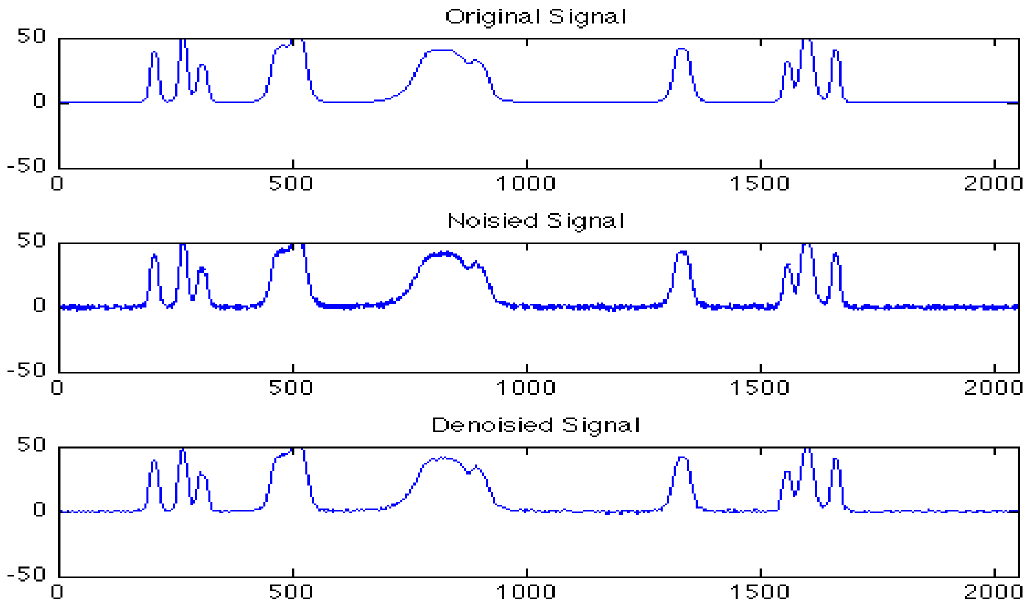


(b)

FIGURE 1 – SID denoising with two iterations for the signal `Blocks`. In (a) original signal. In (b) the corrupted signal by Gaussien noise. In (c), the denoised signal obtained after the first iteration with their associated signal-to-noise ratio (SNR) 23, 46 in dB.



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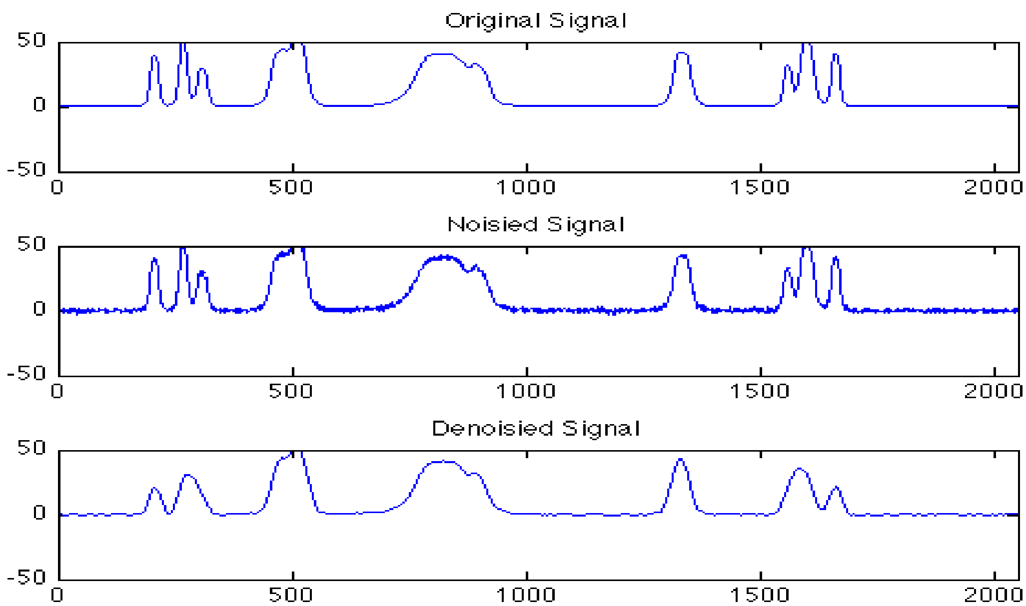
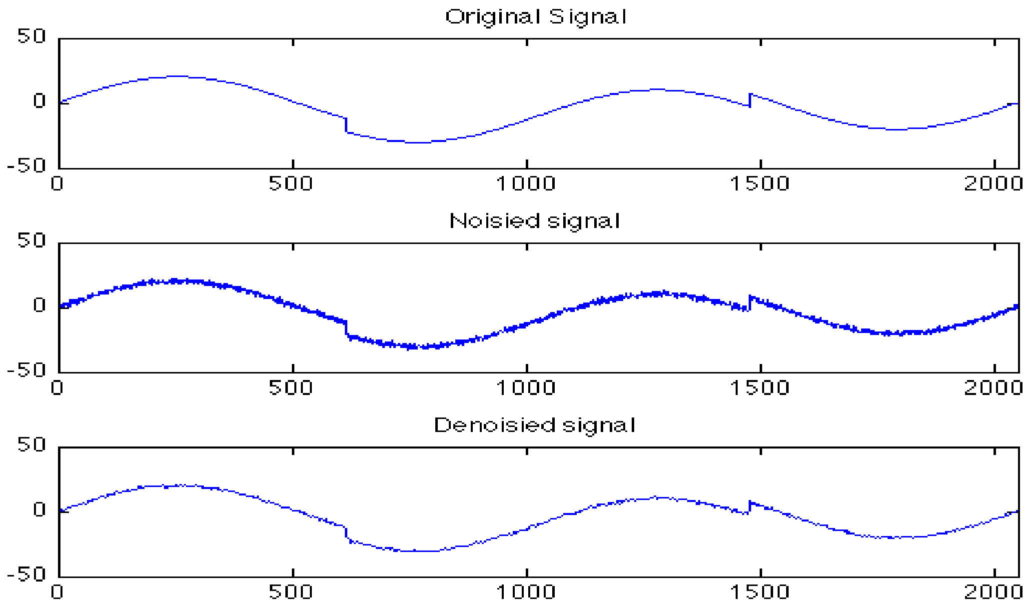


(b)

FIGURE 2 – SID denoising for `wave form` signal with $p = 2$. In (a) original signal. In (b) the corrupted signal by Gaussien noise. In (c), the denoised signal obtained after the first iteration with their associated signal-to-noise ratio (SNR) $23,82$ in dB.



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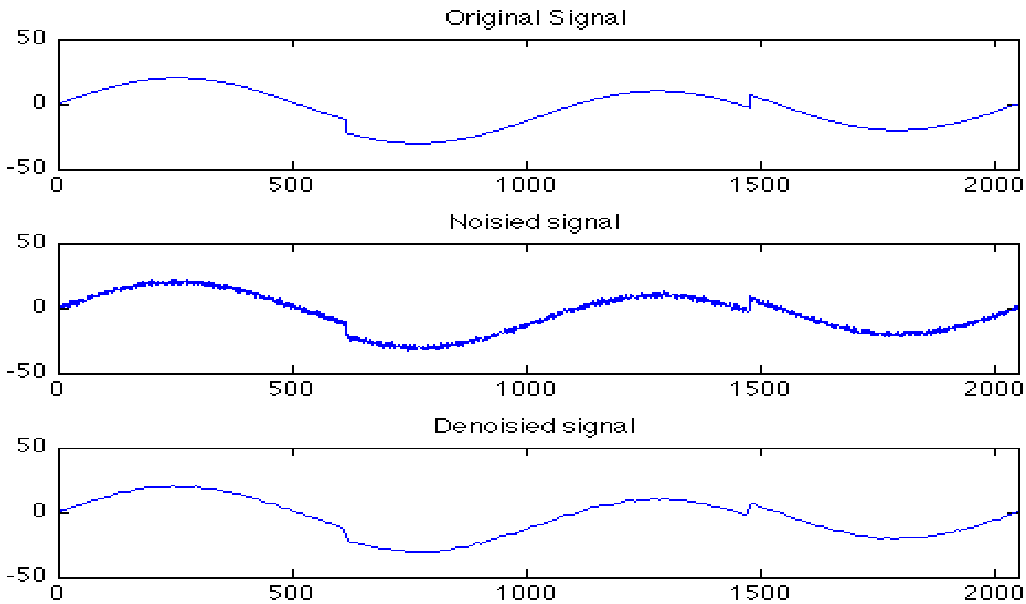

(b)

FIGURE 3 – SID denoising for `Heavy-Sine` signal with two iterations. In (a) original signal. In (b) the corrupted signal by Gaussien noise. In (c), the denoised signal obtained using after two iterations with the associated signal-to-noise ratio (SNR) 29, 72 in dB.