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Study of the Nonlinear Physicals Systems by Optimal Derivative. Applications

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Abstract :
The aim of this work is to present the survey of the computational mathematics method called Optimal Derivative interoduced by Arino-Benouaz(1995, 1996, 2000) and develloped by Bohner_Benouaz (2007, 2009, 2011) using in the modelling of the nonlinear physicals systems.
Keywords: O*ptimal derivative , Computational procedure, Stability, Nonlinear physical system, Classical linearization.*

1. Introduction

The study of differential equations is a mathematical field that has historically been the subject of much research, however, continues to remain relevant, by the fact that it is of particular interest in such disciplines as engineering, physical sciences and more recently biology and electronics, in which many models lead to equations of the same type. Most of these equations are generally nonlinear in nature. The term “nonlinear” gathers extremely diverse systems with little in common in their behavior. It follows that there is not, so far, a theory of nonlinear equations. A large class of these nonlinear problems is modelled by nonlinear ordinary differential equations. Linearization methods play an important role in the analysis of ordinary differential equations. A classical linear approximation is obtained by the Frechet derivative of a nonlinear equation.

Most analytical methods for obtaining approximate solutions to nonlinear ordinary differential equations require that the nonlinearities be suffciently differentiable in order to determine higher-order approximations. Such methods include perturbation techniques [1,4], standard and modified Linsted Poincare procedures [5,7], Adomian’s decomposition method [8,10], the homotopy analysis technique [11,12], the homotopy perturbation method [7] which is a special case of the homotopy analysis technique and is obtained from the latter by setting the parameter that is used to increase the convergence radius to one (the homotopy perturbation method also corresponds to the differential form of Adomian’s decomposition method), artificial parameter techniques [7,17,22], iterative linearized and quasi linearized harmonic balance methods [23,26], etc.
In fact, a careful study of the above techniques indicates that they require that the nonlinearities be analytic functions of the dependent variables and their derivatives with respect to the independent variable. When this is not the case, e.g., when there are fractional-power nonlinearities, one may provide some higher-order approximations by introducing generalized functions [27,32] or the theory of distributions [33] and employing weak convergence [34]. However, even when generalized functions are used, one frequently has to deal with the presence of monopoles, dipoles, quadrupoles, etc., which correspond to the Dirac delta function and its first and second-order derivatives, etc., respectively, whose Fourier series do not converge point-wise.

The study of stability of the equilibrium point of a nonlinear ordinary differential equation is an almost trivial problem if the function F which defines the nonlinear equation is suffciently regular in the neighborhood of this point and if its linearization in this point is hyperbolic. In this case, we know that the nonlinear equation is equivalent to the linearized equation, in the sense that there exists a local diffeomorphism which transforms the neighboring trajectories of the equilibrium point to those neighbors of zero of the linear equation. On the other hand, the problem is all other when the nonlinear function is nonregular or the equilibrium point is the center.

Consider the nonregular case. Imagine the case when the only equilibrium point is nonregular. In this case, we cannot derive the nonlinear function and consequently we cannot study the linearized equation. A natural question arises then: Is it possible to associate another linear equation to the nonlinear equation which has the same asymptotic behavior ?

The idea proposed by Benouaz and Arino is based on the method of approximation. In [39, 40], the authors introduced the optimal derivative, which is in fact a global approximation as opposed to the nonlinear perturbation of a linear equation, having a distinguished behavior with

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respect to the classical linear approximation in the neighborhood of the stationary point. This technique presented in the paper do not require the derivatives of the nonlinearities (neither do they require the presence of small parameters in the ordinary differential equations) and are, therefore, applicable to nonlinear oscillators with non-smooth nonlinearities.

The aim of this paper is to present several examples using the optimal derivative. After a brief review of the optimal derivative procedure in the second section, the third section is devoted to the study of the relationship between the optimal derivative and Frechet derivative in the equilibrium point in the scalar and vectorial case. In the fourth section , we prove for a class of functions that the optimal derivative can be computed even though the classical linearization in 0 does not exist. In the last section, we present two applications in relation with the problem in Electronic and mecanical systems , the study shows, in particular, the influence of the choice of initial conditions. A comparizon with the classical linearization

2. The Optimal Derivative

2.1. The Method

Consider a nonlinear ordinary differential problem of the form:

$$\frac{dx}{dt} = F(x), \quad x(0) = x_0 \tag{1}$$

Where

- $x = (x_1,...,x_n)$ is the unknown function,
- $F = (f_1,....,f_n)$ is a given function on an open subset $\Omega \subset R^n$,

with the assumptions

(H1) $F(0) = 0$,

(H2) the spectrum $\sigma(DF(x))$ is contained in the set {z: Rez < 0} for every $x \neq 0$ in a neighborhood of 0, for which DF(x) exists.

(H3) F is γ Lipschitz continuous.

Consider $x_0 \in IR^n$ and the solution x of the nonlinear equation starting at x0. With all linear $x_0 \in A(L(R^n))$, we associate the solution y of the problem

$$\frac{dy}{dt} = A\, y(t), \quad y(0) = y_0,$$

and we try to minimize the functional

$$G(A) = \int_0^{+\infty} \| F(y(t)) - A\, y(t) \|^2 \, dt \tag{2}$$

along a solution y. We obtain

$$\tilde{A} = \left[\int_0^{+\infty} [F(x(t))][x(t)]^T \, dt \right] \times \left[\int_0^{+\infty} [x(t)][x(t)]^T \, dt \right]^{-1} \tag{3}$$

Precisely, the procedure is defined by the following scheme: Given x_0, we choose a first linear map. For example, if F is differentiable in x_0, then we can take $A_0 = DF(x_0)$ or the derivative value in a point in the vicinity of x_0. This is always possible if F is locally Lipschitz. If A_0 is an asymptotically stable map, then the solution starting from x_0 of the problem.

$$\frac{dy}{dt} = A_0\, y(t), \quad y(0) = y_0,$$

tends to 0 exponentially. We can evaluate G(A) using criteria and we minimize G for all matrices A. If F is linear, then the minimum is reached for the value A = F (and we have A_0 = F). Generally, we can always minimize G, and the matrix which gives the minimum is unique. We call this matrix A_1 and replace A_0 by A_1, we replace y by the solution of the linearized equation associated to A_1, and we continue. The optimal derivative A is the limit of the sequence build as such (for details see [39]).

2.2 Properties of the Method

We will now consider situations where the procedure converges.

- Influence of the choice of the initial condition

Note that if we change x(t) to z, then the relation atilda can be written as

$$\tilde{A} \oint_0^{x_0} z dz^T = \int_0^{x_0} F(z) dz^T ,$$

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where $\oint_0^{x_0}$ is the curvilinear integral along the orbit $\gamma(x_0)= \{e^{Bt} : t \geq 0\}$ of x_0. We obtain

$$\tilde{A} = \left(\oint_0^{x_0} F(z) dz^T \right) \left(\oint_0^{x_0} z dz^T \right)^{-1}$$

It is clear that the optimal derivative depends on the initial condition x_0.

2.2.1 Case when *F* is linear

If F is linear with σ(F) in the negative part of the complex plane, then the procedure gives F at the first iteration. Indeed, in this case, (3) reads

$$A\,\Gamma(x) = F\,\Gamma(x)$$

and it is clear that A = F is a solution. It is unique if $\tilde{A}(x)$ is invertible. Therefore, the optimal approximation of a linear system is the system itself.

2.2.2 Case when F is the sum of a linear and nonlinear term

Consider the more general system of nonlinear equations with a nonlinearity of the form

$$F(x) = M\,x + \tilde{F}(x), \qquad x(0) = x_0,$$

where M is linear. The computation of the matrix A_1 gives

$$A_1 = \left[\int_0^{+\infty} [F(x(t))][x(t)]^T\,dt \right] [\Gamma(x)]^{-1}$$

$$= \left(M\,\Gamma(x) + \int_0^{+\infty} \left[\tilde{F}(x(t))\right][x(t)]^T\,dt \right) [\Gamma(x)]^{-1}$$

$$= M + \left(\int_0^{+\infty} \left[\tilde{F}(x(t))\right][x(t)]^T\,dt \right) [\Gamma(x)]^{-1}$$

Hence, $A = M_1 + \tilde{A}_1$ with

$$\tilde{A}_1 = \left[\int_0^{+\infty} \left[\tilde{F}(x(t))\right][x(t)]^T\,dt \right] [\Gamma(x)]^{-1}$$

Then, for all j we have $A_j = M + \tilde{A}_j$ with

$$\tilde{A}_j = \left[\int_0^{+\infty} \left[\tilde{F}(x_j(t))\right][x_j(t)]^T\,dt \right] [\Gamma(x_j)]^{-1}$$

If, in particular, some components of F are linear, then the corresponding components of $\tilde{F}$ are zero, and the corresponding components of A_j are those of F. If f_k is linear, then the k_{th} row of the matrix A_j is equal to f_k.

3. Relationships Between the Optimal Derivative and the Classical Linearization in Zero

3.1. Scalar case

3.1.1 Expression

Consider the scalar differential problem

$$\frac{dx}{dt} = f(x), \quad x(0) = x_0 \tag{4}$$

with $f : R \to R$ and under the assumptions

(h_1) $f(0) = 0$,

(h_2) $f'(x) < 0$ in every point where f' exists in an interval $(-\alpha, \alpha)$ with $\alpha > 0$,

(h_3) f is absolutely continuous with respect to the Lebesgue measure.

The calculation is done in a way similar to that of the vectorial case. We start with the calculation of $a0 = f'(x_0)$, then we calculate a_1 by solving the problem

$$\frac{dx}{dt} = a_0\,x, \quad x(0) = x_0$$

By changing F to f in (3), we have

$$a_1 = \frac{\int_0^{+\infty} f(x(t))\,x(t)\,dt}{x_0 \int_0^{+\infty} x^2(t)\,dt},$$

and by substituting $x = \exp(a_0\,t)\,x_0$, we obtain

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$$a_1 = \frac{\int_0^{x_0} f(x)\,dx}{\int_0^{x_0} x\,dx} = \frac{2}{x_0^2}\int_0^{x_0} f(x)\,dx$$

Note that a_1 does not depend on a_0, and consequently, the procedure for the optimal derivative converges in the first step, namely

$$\tilde{a} = \tilde{a}(x_0) = \frac{2}{x_0^2}\int_0^{x_0} f(x)\,dx \qquad (5)$$

We remind the reader that it has been shown that $\tilde{a}(x_0)$ is a Lyapunov function [40] for the nonlinear problem (4). The scalar case is very interesting in the sense that we can write the optimal derivative as a function of the classical linearization of f in 0 (if $f^{/}$ exists in 0); so it is possible to find a limit when $x_0 \to 0$, namely $\tilde{a}(x_0)$, even though the derivative of f in 0 does not exist. The importance of the result lies in the possibility of using $\tilde{a}(x_0)$ for the description of the behavior of the solution and for the study of stability in the vicinity of 0 when the derivative in this point does not exist.

3.1.2 Case when the derivative of f in 0 exists

If f is continuous and if the derivative of f in 0 exists, then it is known [?] that $\tilde{a}(x_0)$ can be written as

$$\tilde{a}(x_0) = f^{/}(0) + \frac{2}{x_0^2}\int_0^{x_0} z\,\varepsilon(z)\,dz\,,$$

where

$$\varepsilon(z) = \frac{f(z)}{z} - f^{/}(0)$$

and that $\lim_{x0\to 0} = \tilde{a}(x_0) = f^{/}(0)$. This relation shows that the two quantities $\tilde{a}(x_0)$ and $f^{/}(0)$ are almost equal and are equal in the limit as x_0 tends to 0.

3.1.3 Case when f is analytic in 0

Now assume that f is analytic in 0, i.e.,

$$f(x) = \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!}\,x^n$$

Then it is possible to give an expansion of $\tilde{a}(x_0)$ similar to the Taylor expansion of f in the neighborhood of 0. For this, we use the relation scaopt and replace f(z) by the expression given by relation (6) so that

$$\tilde{a}(x_0) = \frac{2}{x_0^2}\int_0^{x_0}\sum_{n=1}^{\infty}\frac{f^{(n)}(0)}{n!}\,x^n\,dx = 2\sum_{n=1}^{\infty}\frac{f^{(n)}(0)}{(n+1)!}\,x_0^{\,n-1}$$
$$= f^{/}(0) + \frac{1}{3}x_0\,f^{/}(0) + \ldots + \frac{2}{(n+1)!}x_0^{\,n-1}\,f^{(n)}(0) + \ldots,$$

Where this formula holds in the interval of convergence of the Taylor series in 0. Generally, if f is of class Ck with $k \in N$ in the vicinity of 0 and f(0) = 0, then $\tilde{a}$ is of class C^{k-1} in this vicinity, and we obtain

$$\tilde{a}^{(j)}(0) = \frac{2}{(j+1)!}x_0^{\,j-1}\,f^{(j)}(0), \qquad 0 \le j \le k-1$$

3.1.4 Case when f is not regular in 0

We now consider the nonregular case, and more particularly the case that f is only nondifferentiable in 0.Writing f(z) in the form

$$f(z) = -\,z\,g(z)\,,$$

the relation (5) becomes

$$\tilde{a}(x_0) = -\frac{2}{x_0^2}\int_0^{x_0} z\,g(z)\,dz \qquad (7)$$

The chosen function

$$g_r(z) = p\left(|\ln z|^r\right),$$

where p is a bounded nonnegative periodic function of period 1 satisfying $\overline{p} = \int_0^1 p(z)\,dz > 0$, is nondifferentiable in 0. The relation (7) is written for r =1 and $0 < x_0 < 1$ as

$$\tilde{a}(x_0) = -\frac{2}{x_0^2}\int_0^{x_0} z\,p(|\ln z|)\,dz$$

For all $\alpha \in (0, 1)$,we have

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$$\tilde{a}(\alpha x_0) = -\frac{2}{\alpha^2 x_0^2}\int_0^{\alpha x_0} z\, p(-\ln z)\, dz$$
$$= -\frac{2}{\alpha^2 x_0^2}\int_0^{x_0} \alpha^2 z\, p(-\ln\alpha - \ln z)\, dz$$
$$= -\frac{2}{x_0^2}\int_0^{x_0} z\, p(-\ln\alpha - \ln z)\, dz$$

So in particular, if ln α = −1, i.e., $\alpha = e^{-1}$, then $\tilde{a}(x_0/e) = \tilde{a}(x_0)$. In this case, $\tilde{a}(x_0)$ does not have limit when $x_0 \to 0^+$. In the case r > 1, we obtain

$$\tilde{a}_r(x_0) = -2\int_0^1 z\, p\{\, (-\ln x_0 - \ln z)^r \,\}\, dz$$

Let us now consider the relation

$$\tilde{a}_r(x_0) = -\frac{2}{x_0^2}\int_0^{x_0} u\, g_r(u)\, du \qquad (8)$$

where $g_r(u) = p(|\ln u|^r)$.Note that $g_r(u)$ is non differentiable in 0. In this case, we will show that the optimal derivative (8) can exist even if the derivative of the function $g_r(u)$ in 0 does not exist. Then

$\tilde{a}_r(x_0) \to -\bar{p}$ when $x_0 \to 0$ for every r > 1.

For more details, see the proof given in [48], Although the stability criteria by linearization are clearly stated and rigorously justified, classical linearization is sometimes inconvenient because it assumes that the Jacobian matrix at the equilibrium point exists. However, this assumption is not always true. Consider for instance a nonlinear system with a function involving an absolute value such that the nonlinearity is not differentiable in the vicinity of the equilibrium point. The classical linearization gives a necessary condition but not a suffcient one, since it does not allow to study stability in the presence of purely imaginary eigenvalues. The search for a Lyapunov function itself constitutes a sensitive issue because it is based in general on experience and luck.

4. Computational Procedure

First of all let us point out briefly the iterative procedure allowing the calculation of the optimal derivative. Starting the calculus, the point x0 is selected arbitrarily near the origin. The differential equations have been solved using the fourth order Runge-Kutta method [16, 19].

- Input x_0 and A_0.
- Level (I): Computation of A_1 in terms of A_0:

$$A_1 = \left[\int_0^{+\infty} F\left(e^{tA_0}x_0\right)\left(e^{tA_0}x_0\right)^T dt\right] \times \left[\int_0^{+\infty} \left(e^{tA_0}x_0\right)\left(e^{tA_0}x_0\right)^T dt\right]^{-1}$$

- Level (II): Computation of A_j in terms of A_{j-1}:

$$A_j = \left[\int_0^{+\infty} F\left(e^{tA_{j-1}}x_0\right)\left(e^{tA_{j-1}}x_0\right)^T dt\right] \times \left[\int_0^{+\infty} \left(e^{tA_{j-1}}x_0\right)\left(e^{tA_{j-1}}x_0\right)^T dt\right]^{-1}$$

- Level (III): Computation of

$$\|A_j - A_{j-1}\|$$

- Level (IV): If

$$\|A_j - A_{j-1}\| < \varepsilon$$

where ε is the desired level of approximation, then set $\tilde{A} = A_j$. $\tilde{A}$ is the optimal derivative of F at x_0. Otherwise set $A_j = A_{j-1}$ and go to Level (II).

Remark 4 The precision of the optimal derivative is expressed in terms of the norm of the initial condition x_0[8] and is given by

$$\|x(t) - \tilde{y}(t)\| < O(\|x_0\|)^2$$

In the previous work, we have show for which initial conditions the precision is maintained. As long as $\|x_0\|$ is large in a certain sense, the approximation must be good. It becomes more difficult when approaching 0. Indeed, it is shown that the approach of 0 yields inversion of the quadratic error to the profit of the classical linearization.
This shows that the classical linearization is better near the origin when it exists. Let us present examples emphasizing the theoretical aspect in relation to the influence of the choice of the initial conditions on the quality of the approximation.

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5 Application

5.1. Introduction

We present two examples, the first from the nonlinear electronic system is presented in detail and comparaison with classical linearization is aborded. Second example is cited with analyse as a reference for possibilities offer by the optimal derivative.

5..2. Fist Example

The function of the electronic circuit (see …) in the Figure 1is represented by two variables of states the voltage drop V_{C1}on the terminal of the first capacity and the voltage drop V_{C2}on the terminal of the second capacity). The

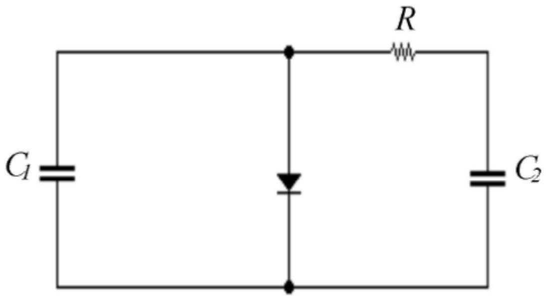

Fig 1: Circuit used in the example,

When a tension $V_C$is applied to the diode in the direct direction, the model of the diode is given by

$$f(V_{C1}) = \begin{cases} 0 & if V_{C1} < 0 \\ aV_{C1} + bV_{C_1}^2 + dV_{C_1}^4 & if V_{C1} \geq 0 \end{cases}$$

With the parameters

$R = 33.10^2\Omega,\quad C_1 = 220.10^{-4}F,\quad C_2 = 350.10^{-4}F,$

$a = 10^{-4}, b = 10^{-5},\qquad d = 10^{-6}$

and starting from the laws of Kirchoff relating to the nodes and the meshes of the circuit, we obtain the equations

$$\begin{cases} \frac{dV_{C1}}{dt} = -\frac{1}{C_1}(aV_{C1} + bV_{C_1}^2 + dV_{C_1}^4 + \frac{V_{C1} - V_{C2}}{R}) \\ \frac{dV_{C2}}{dt} = \frac{1}{RC_2}[V_{C1} - V_{C2}]. \end{cases}$$

Changing

$$x = V_{C1} and\ y = V_{C2},$$

The system (eqref) can be rewritten as

$$\begin{cases} \frac{dx}{dt} = -\frac{a}{C_1}x - \frac{b}{C_1}x^2 - \frac{d}{C_1}x^4 - \frac{1}{RC_1}x + \frac{1}{RC_1}y \\ \frac{dy}{dt} = \frac{1}{RC_2}x - \frac{1}{RC_2}y. \end{cases}$$

By replacing the parameters with their values, the system becomes

$$\begin{cases} \frac{dx}{dt} = -(1.8.10^{-2}x + 4.55.10^{-5}(10x^2 - x^4) - 1.38.10^{-2}y) \\ \frac{dy}{dt} = 8.66.10^{-3}(x - y). \end{cases}$$

- Classical linearization

The classical linearization at the equilibrium point (0, 0) is obtained by calculation the Frechet derivative of the nonlinear function of the system (eqref),

$$DF(0,0) = \begin{bmatrix} -1.8.10^{-2} & 1.38.10^{-2} \\ 8.66.10^{-3} & -8.66.10^{-3} \end{bmatrix}.$$

- Optimal derivative

The optimal derivative is obtained by applying the algorithm proposed above, see section (ref). For the quadratic error, we use the relation

$$E_Q = \sum_{i=1}^{n} \|x_i(t) - \tilde{y}_i(t)\|^2,$$

Where

$x(t)$ represents a solution of the nonlinear system,

$y(t)$ represents a solution of the optimal derivative.

5.2.1 Results of the method

We study the system using several initial conditions. The results obtained are exhibited in the Table 1, where E_{Qmax} (O.D.) and E_{Qm}(C.L.) represent the maximum quadratic errors for the optimal derivative and the classical linearization, respectively. In the left column the initial conditions (x_0, y_0) are given. The second column represents the optimal derivative $\tilde{A}$.

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Table 1

(x_0, y_0)	$\tilde{A}$	E_{Qm}(O.D.)	E_{Qm}(C.L.)
(0.8,0.5)	$\begin{matrix} -0.0187 & 0.0142 \\ 0.0087 & -0.0087 \end{matrix}$	2.1302e-04	3.5140e-04
(8e-02,5e-01)	$\begin{matrix} -0.0181 & 0.0138 \\ 0.0087 & -0.0087 \end{matrix}$	7.5438e-06	1.0367e-05
(8e-02,5e-02)	$\begin{matrix} -0.0181 & 0.0138 \\ 0.0087 & -0.0087 \end{matrix}$	7.4729e-09	2.2644e-08
(8e-03,5e-02)	$\begin{matrix} -0.0180 & 0.0138 \\ 0.0087 & -0.0087 \end{matrix}$	8.5925e-10	1.0691e-09
(8e-03,5e-03)	$\begin{matrix} -0.0180 & 0.0138 \\ 0.0087 & -0.0087 \end{matrix}$	7.0425e-13	2.2132e-12
(8e-04,5e-03)	$\begin{matrix} -0.0179 & 0.0138 \\ 0.0087 & -0.0087 \end{matrix}$	9.0836e-14	1.0969e-13
(8e-04,5e-04)	$\begin{matrix} -0.0179 & 0.0138 \\ 0.0087 & -0.0087 \end{matrix}$	2.2657e-17	1.3572e-16
(8e-05,5e-05)	$\begin{matrix} -0.0178 & 0.0138 \\ 0.0087 & -0.0087 \end{matrix}$	3.249e-21	3.481e-21

The curve

$$E_{Qmax} = h(\|x_0\|)$$

In Figure 2 is obtained starting from a smoothing polynomial using the Origin Software. The determination of the value $x_0$for which the curve of error changes behavior will be calculated is performed using the Matlab Softwar.

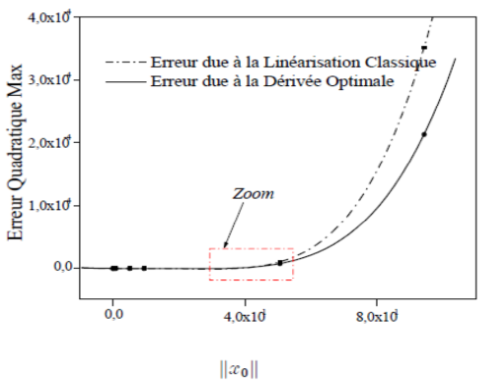

Fig 2: Max quadratic error with respec

In figure 3, a zoom of the part where there is inversion of the quality of the approximation to the profil of the classicallinearization is represented.

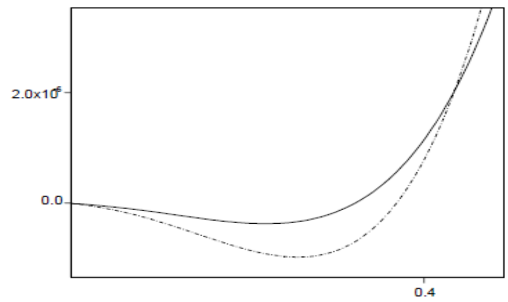

Fig 3: Zoom of the part where there is inversion of the quality of the approximation

5.2.2. Analysis of results of the first example

The representation of the maximum quadratic error with respect to $\|x_0\|$ relating to the classical linearization and the optimal derivative enables us to divide our curve into two distinct parts:
The first part, where the maximum quadratic error due to the classical linearization is lower than that due to the optimal derivativon an interval of $\|x_0\| < 0.43$ in this case the classical linearization gives a better aproximation than the optimal derivative.

The second part where the maximum quadratic error due to the classical linearization becomes definitely higher than that due to the optimal derivative on an interval of $\|x_0\| > 0.43$ here it is the optimal derivative which is better. Namely, for a given initial condition x_0, approximation by the optimal derivative is better in the vicinity of the origin. These two aspects reflect the fact that the linearization by Frechet derivative (when it exists and when it is hyperbolic) is the best approximation in the vicinity of the origin.

5.3 Second example

This example is derived from a Nonlinear mechanical system representing a forced nonlinear oscillator [50],[51] [52], in fact is a mechanical posistioning device with feed-back control. given by the system:

$$\ddot{x} + \delta\dot{x} + K(x)x = -z + F(t)$$
$$\dot{z} + \alpha z = \alpha\gamma(x - r)$$

x is defined as the displacement, $\delta\dot{x}$ the linear damping with a damping constant δ> 0, object of negative feed-back control (z) with time constant $1/\alpha$ and the gain γ) ,

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$$K(x) = (x^2 - 1)$$

We take $F(t) = 0$, $r = 0$ representing autonomous system, in which the governing equation have no explicit time dependence. Our goal that this type of dynamical systems can still exhibit complicated dynamics (complex bifurcations and transient to chaos) with a regime in with two or more stables limit cycles exist :

$$\begin{aligned} \ddot{x} + \delta\dot{x} + x^3 - x &= -z \\ \dot{z} + \alpha z &= \alpha\gamma x \end{aligned} \tag{4}$$

the system (4) with the dimensionless equation is given by

$$\begin{cases} \dot{x} = y \\ \dot{y} = x - x^3 - \delta y - z \\ \dot{z} = \gamma\alpha x - \alpha z \end{cases} \tag{5}$$

$(x, y, z) \in IR^3$ with the parameters $\delta, \alpha, \gamma > 0$

5.3.1. Discussion

we can deduce from the determination of the bifurcation surfaces that the higher codimension bifurcations can easily be spotted, once the full parameter dependence of the bifurcation surfaces is known.

We can note that the proposed method may be more efficient in term of approximation the nonlinear function is no regular or the equilibrium point is no regular. In this case, one cannot derive the nonlinear function and consequently one cannot study the linearized equation see [37]. In contrast to common analytical techniques based on eigenvalue computation (which can only be applied to systems of size dimension $N \leq 4$), the method is applicable for systems of intermediate size because it is possible to compute numerically the optimal linear matrix and the roots of their characteristic equation (eigenvalues). the proposed linearization representing also a numerical confirmations of the prediction behaviour. Therefore it represent a good approximation to the initial nonlinear system.

6. Conclusion

Simplification is very important in modelling. The optimal derivative procedure can be used as a powerful tool for modelling nonlinear physicals systems numerically. The optimal derivative method helps to give a quantitative and qualitative description of Systems which appear in the behavior of the electronic and mecanical problems.

In conclusion, the answer to the question relative to the relation between the property of stability of the linear equation obtained by the optimal derivative and that of the nonlinear equation in the vectorial case is very subtle. Generally, when the procedure converges, the matrix obtained is stable. All these considerations bring us to the following conjecture.

Conjecture: If the procedure of the optimal derivative converges and the limit of the sequence A_j is exponentially stable (or if A_j has a stable fixed point), then the nonlinear system is stable.

This study shows that the conditions under which the conjecture was formulated can be satisfied, i.e., the existence, uniqueness and convergence towards a stable fixed point [7]. The procedure of calculation also enables us to solve problems where the classical linearization may not be useful.

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